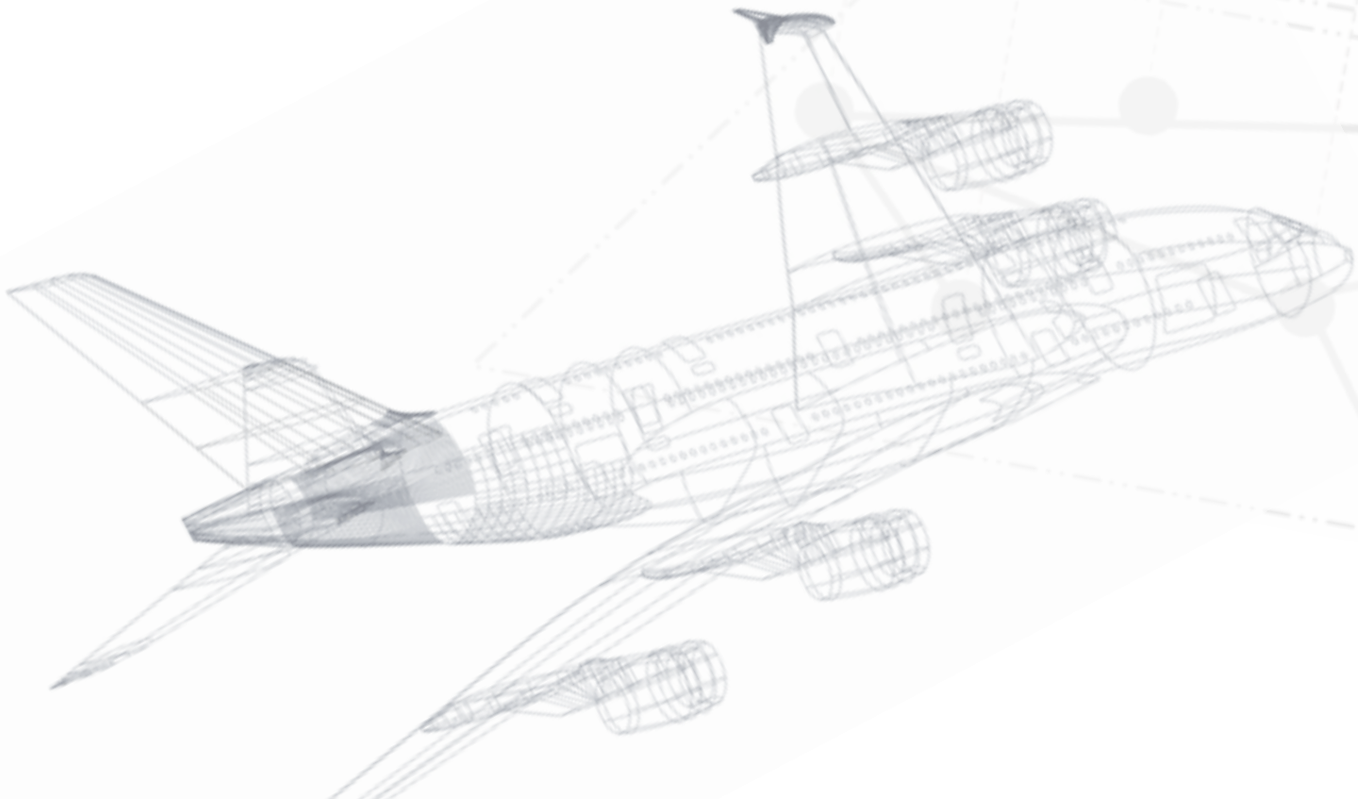
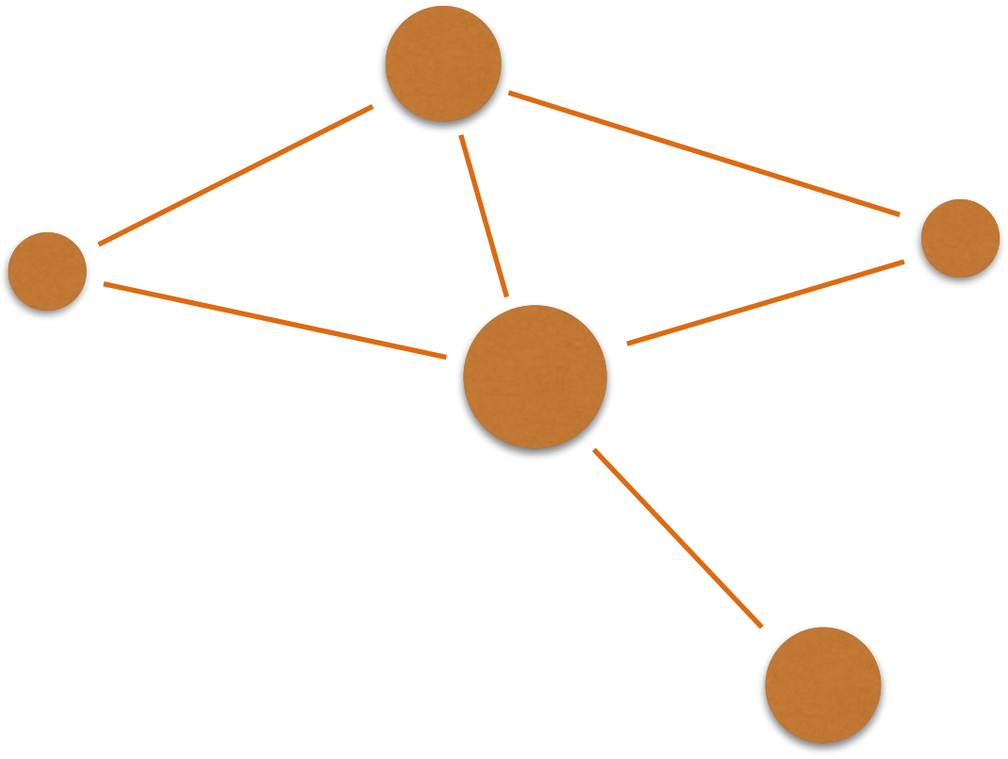
Multi-layer functional networks:

Have we got the basics right?



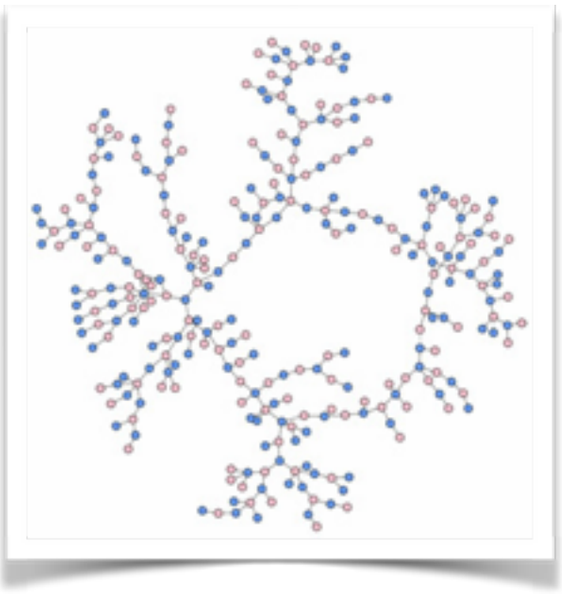
Massimiliano Zanin

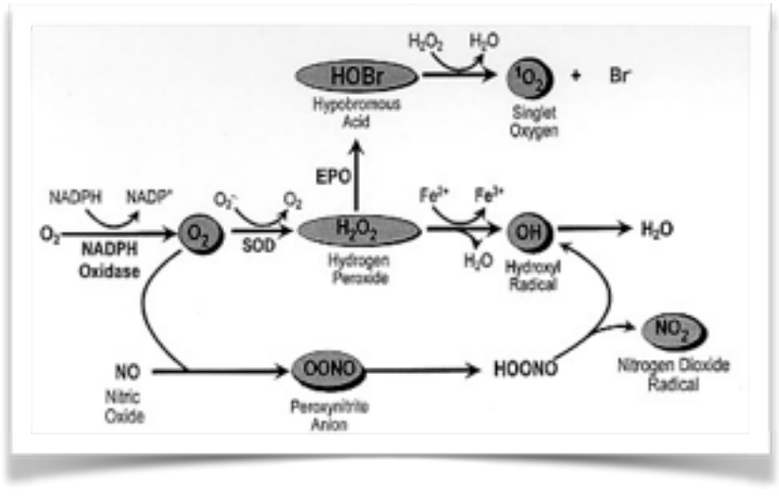
The story so far …



1998:

Birth of complex network theory





Analysis of the Air Transport network

Country	Period	Nodes	Links	γ	γ_B	L	L_{rand}	C	C_{rand}	Refs.
World	11/2000	3883	27051	1.0	0.9	4.4	—	0.62	0.049	[34]
World	11/2002	3880	18810	2.0	—	4.37	—	—	—	[35,36]
US	—	215	*116725	2.0	—	1.403	—	0.618	0.065	[37]
US	10-12/2005	272	6566	2.63	—	1.9	1.81	0.73	0.19	[38]
Austria	—	134	9560	2.32	—	—	—	0.206	0.01	[22]
China	—	128	1165	4.161	—	2.067	—	0.733	—	[23]
China	28/11/2007–29/3/2008	144	1018	—	—	2.23	1.88	0.69	0.098	[39]
India	12/1/2004	79	442	2.2	—	2.259	2.493	0.657	0.0731	[40]
India	12/2010	84	*13909	0.71	0.54	2.17	2.55	0.645	0.18	[41]
Italy	16/7-14/8/2005	42	—	1.6	0.4	1.987	3.74	0.10	0.17	[42]
Italy	11/2005	42	—	1.1	0.5	2.14	3.64	0.07	0.14	[42]
Italy	6/2005–5/2006	42	310	1.7	0.4	1.97	—	0.1	—	[43]
Italy	—	33	105	—	—	1.92	—	0.418	—	[44]
Spain	—	35	123	—	—	1.84	—	0.738	—	[44]

Country	Weigth	β	β_b	θ	Refs.
Worldwide	Available seats	1.5	0.8	0.5	[35,36,49]
US	Number of passengers	1.8	—	—	[38]
India	Number of flights	1.43	—	—	[40]
4 European airlines	Number of flights	$(1.06 - 1.18)$	—	—	[50]
China	Number of flights	—	—	0.5	[23]
Europe	Number of flights	1.39	—	—	[51]

M. Zanin and F. Lillo

Modelling the air transport with complex networks: A short review

Eur. Phys. J. Special Topics 215, 5–21 (2013)

The first problem:

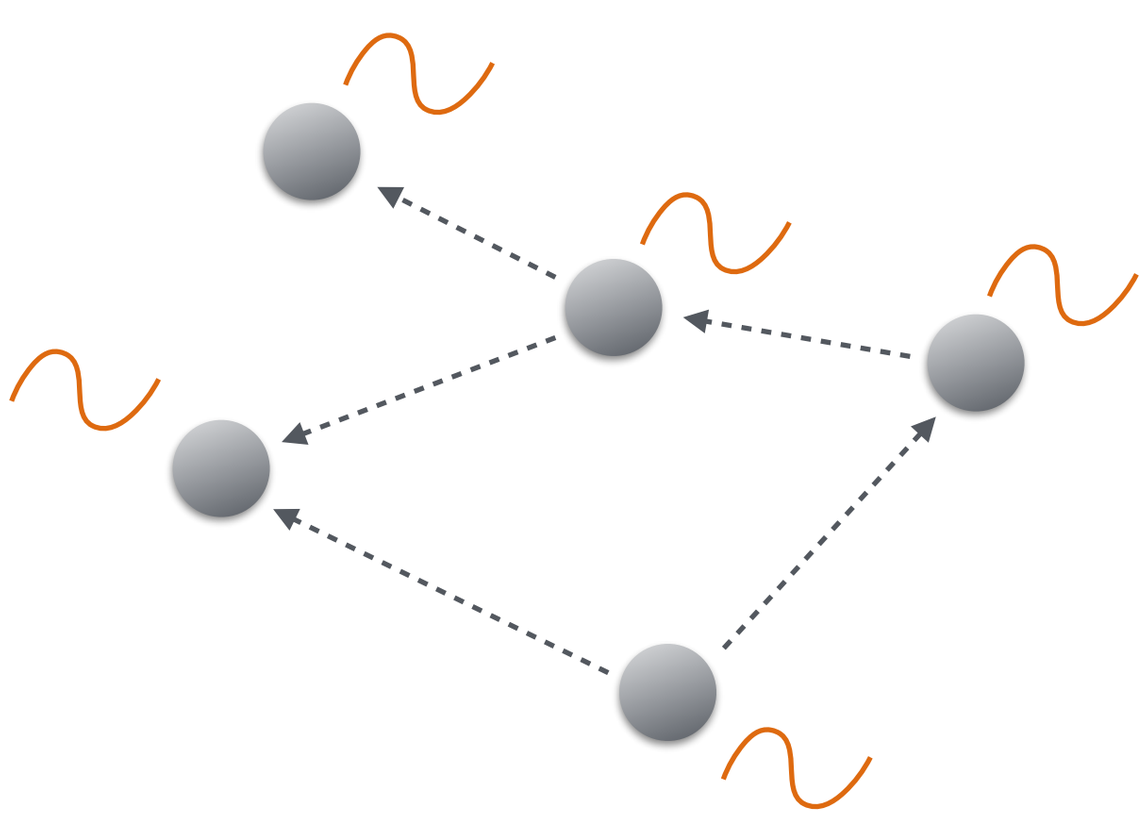
the structure is not always explicit (or measurable)

the *hardware* structure may not be relevant

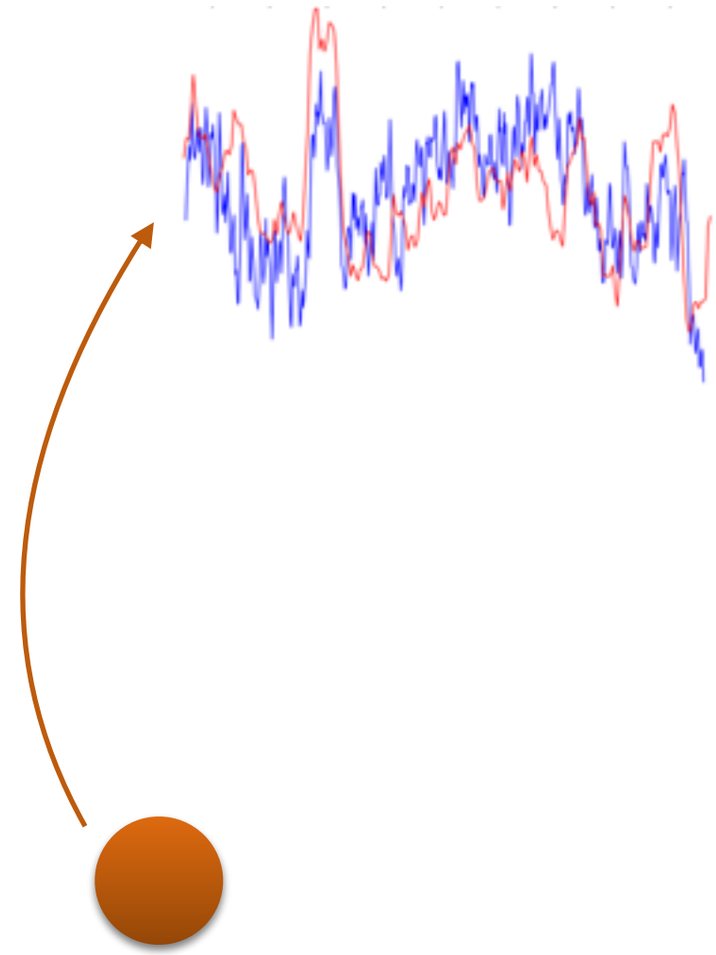
A classical example:

2000:

Birth of functional representations

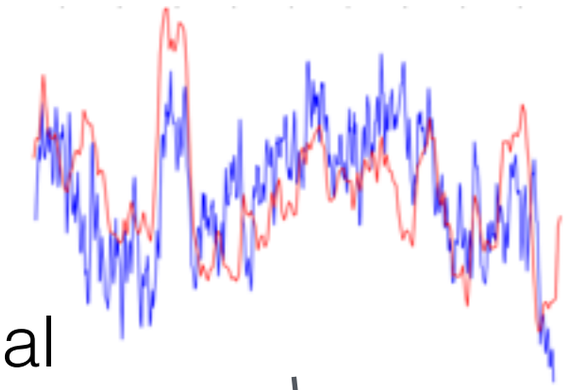


Birth of functional representations



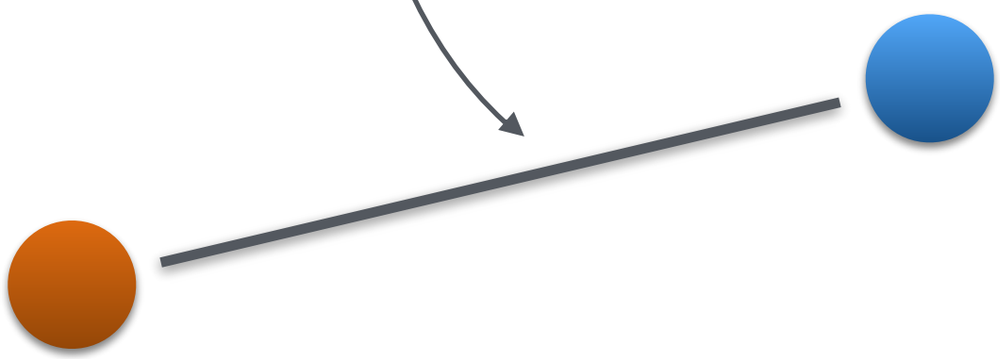
1. Extraction of time series

Birth of functional representations



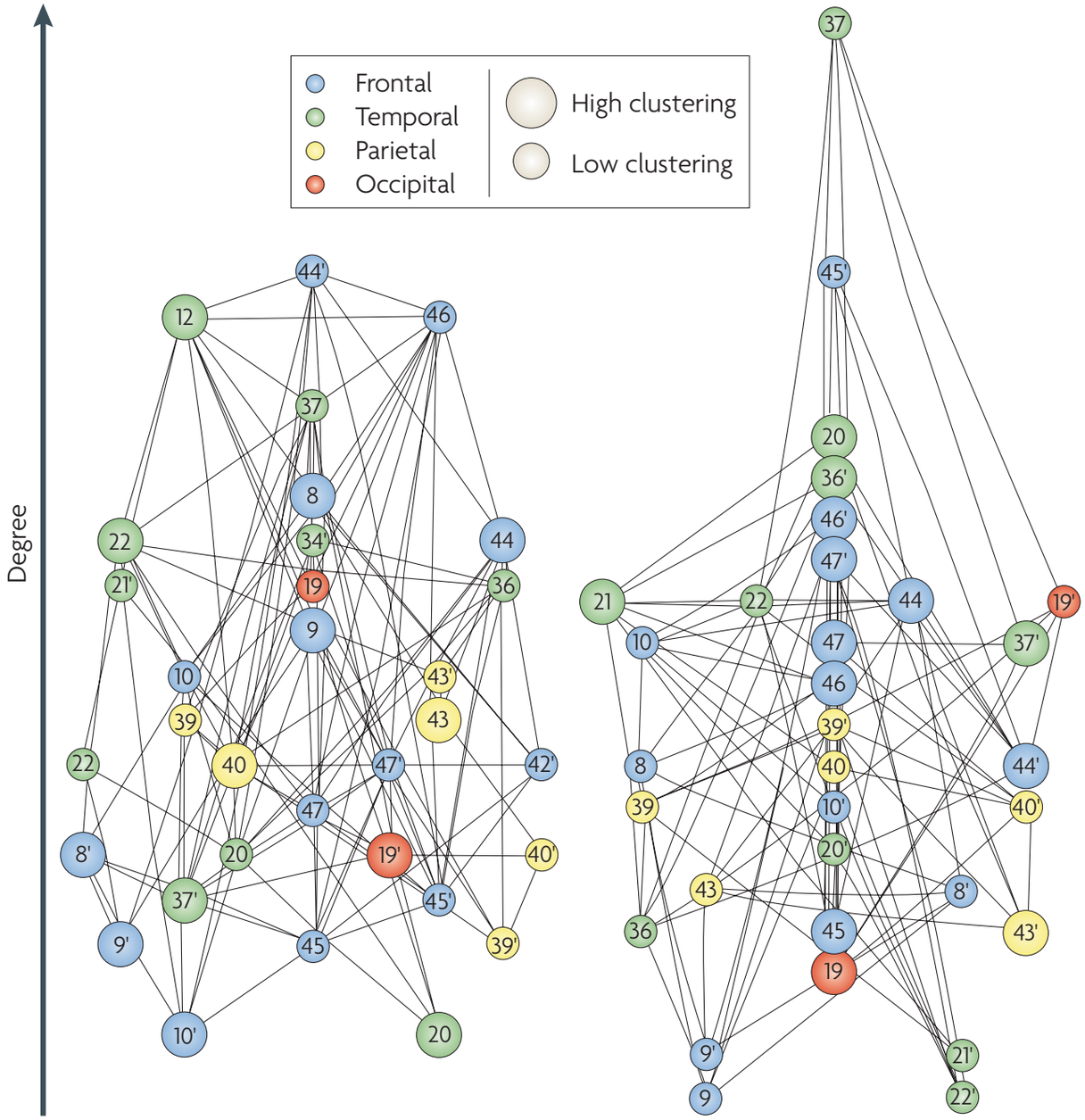
2. Analysis of functional connectivity

Linear correlation
synchronisation
causality
…



a Healthy volunteers

b People with schizophrenia

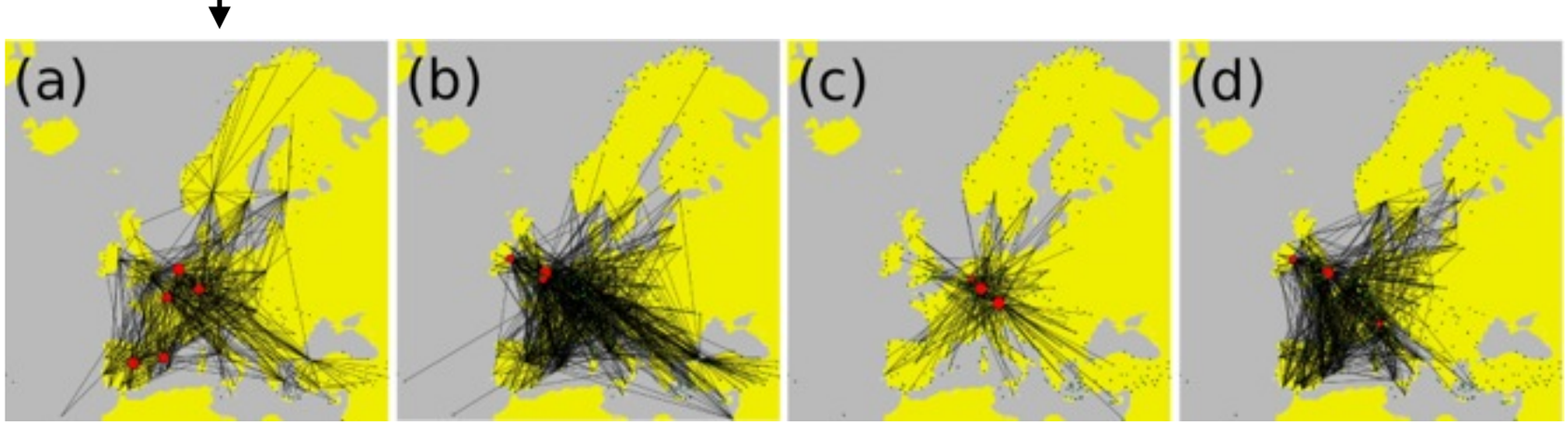


Ed Bullmore and Olaf Sporns
Complex brain networks: graph theoretical analysis of structural and functional systems
Nat. Rev. Neuroscience 10 (2009)

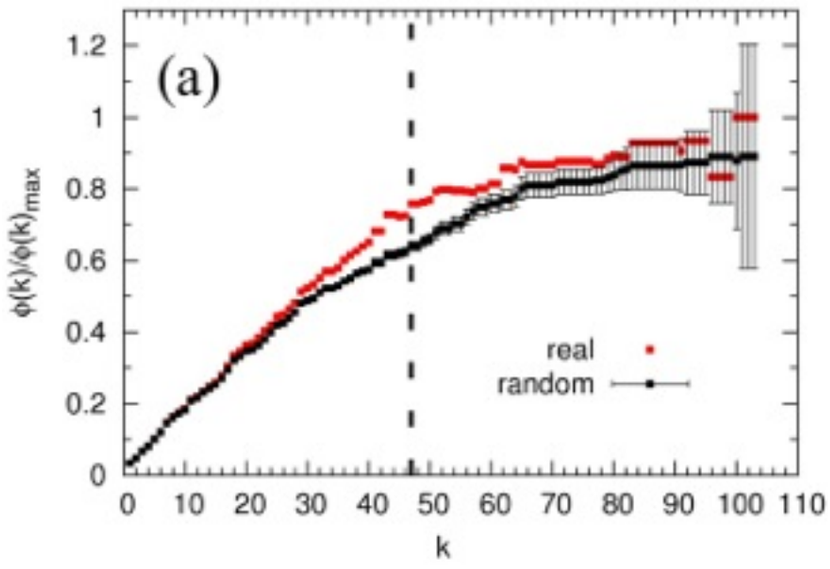
What about multi-layer structures?

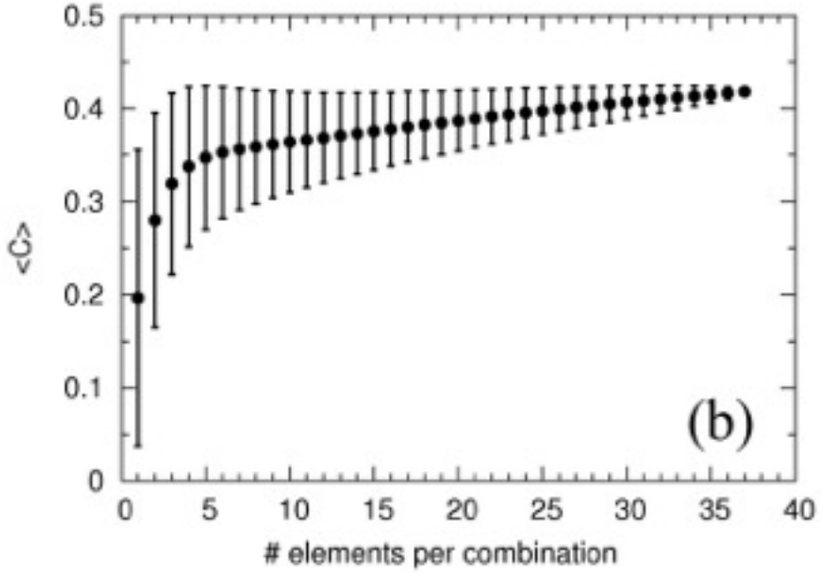
We usually analyse the projected networks …

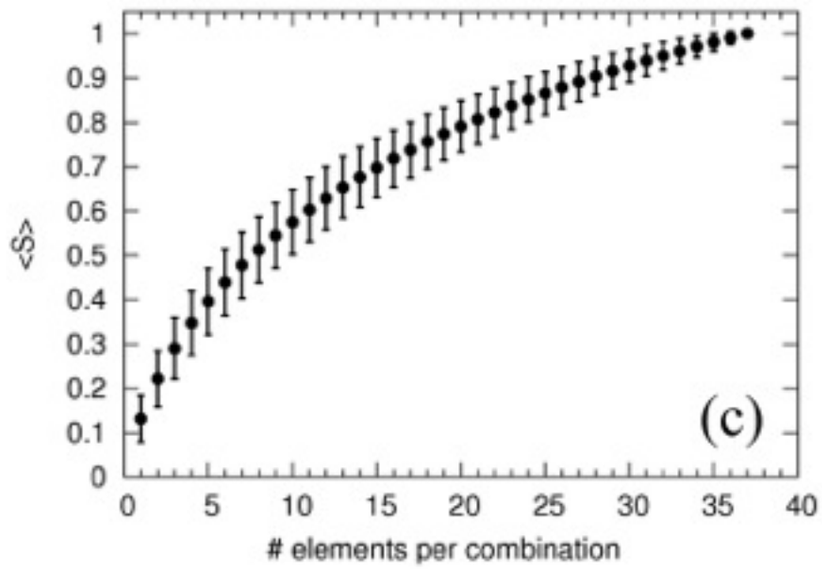
2010:

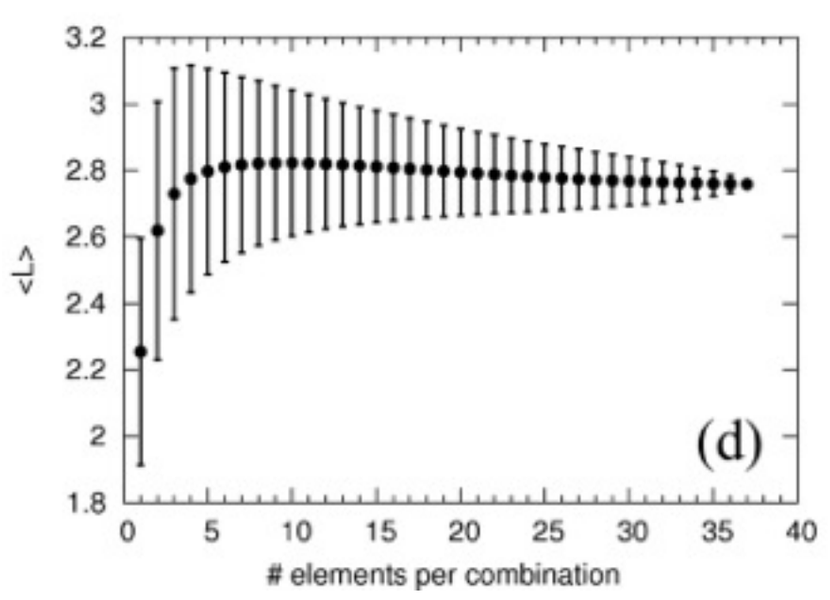


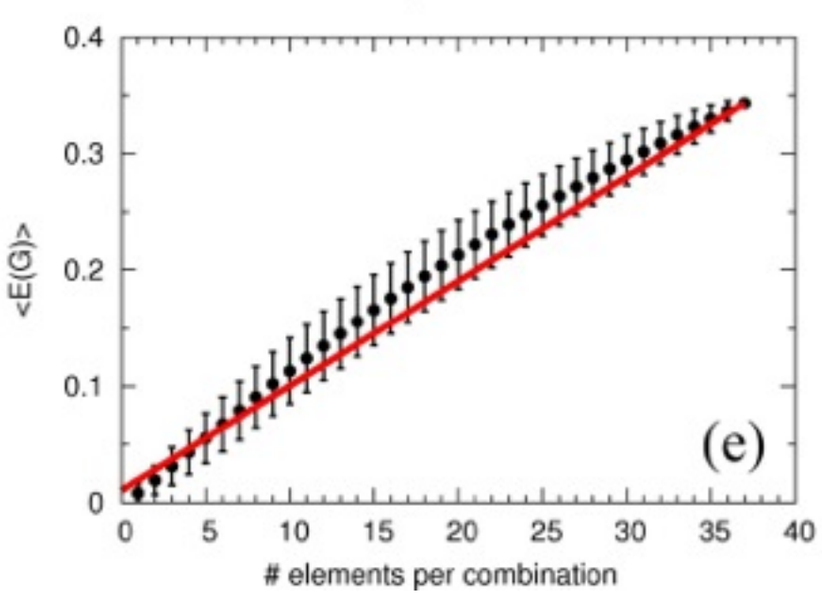
… but the real network is created by airlines

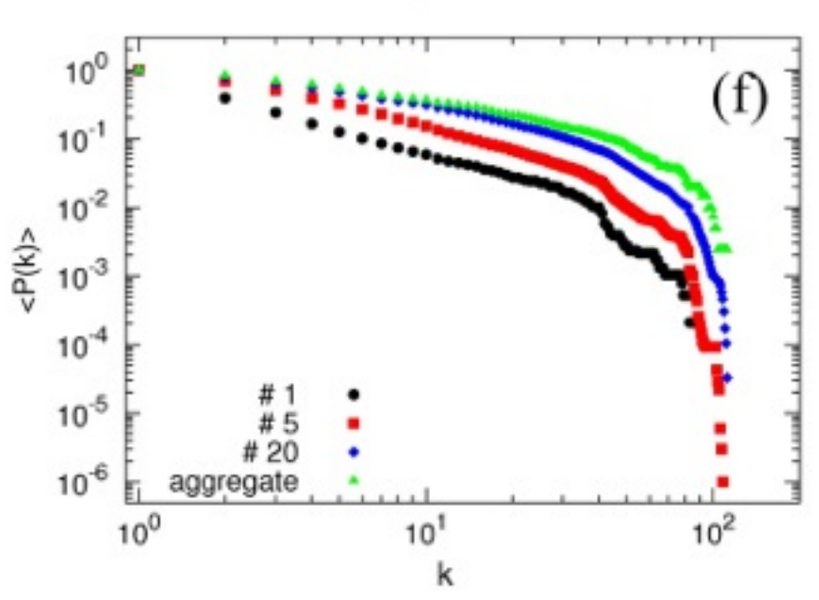




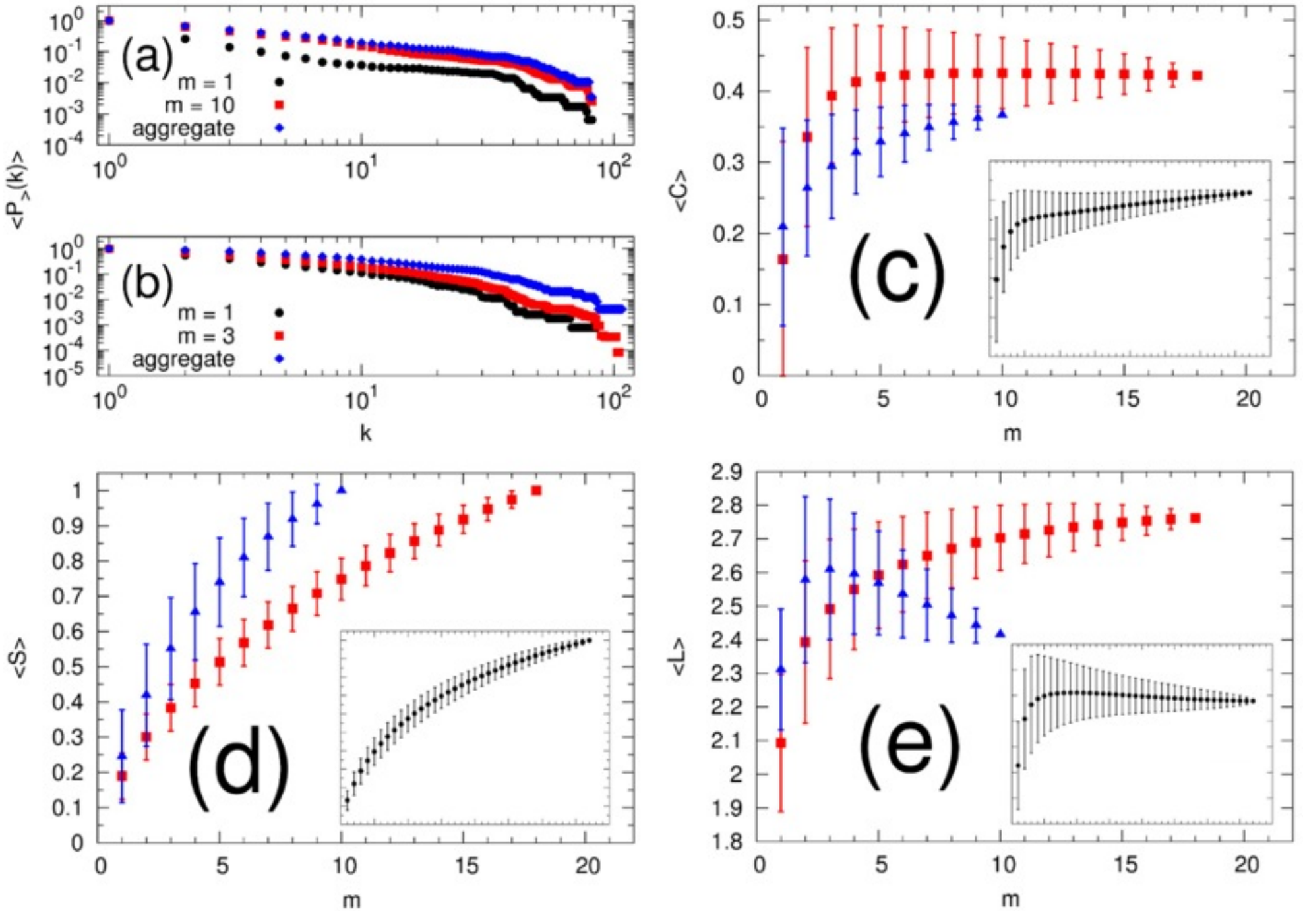








A. Cardillo, J. Gómez-Gardeñes, M. Zanin *et al.*
Emergence of network features from multiplexity
Scientific Reports 2 (2013)



A. Cardillo, J. Gómez-Gardeñes, M. Zanin *et al.*
Emergence of network features from multiplexity
Scientific Reports 2 (2013)

The structure and dynamics of multilayer networks

S. Boccaletti[a,b,*], G. Bianconi[c], R. Criado[d,e], C.I. del Genio[f,g,h], J. Gómez-Gardeñes[i], M. Romance[d,e], I. Sendiña-Nadal[j,e], Z. Wang[k,l], M. Zanin[m,n]

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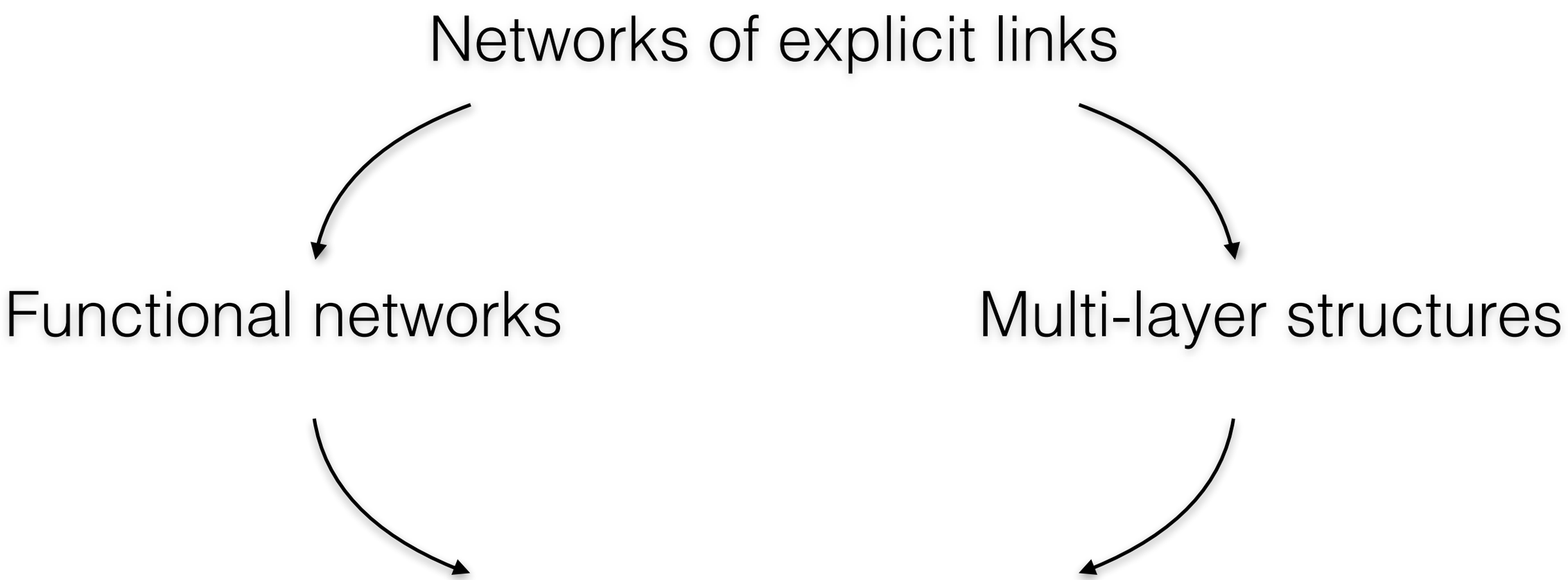
Abstract

In the past years, network theory has successfully characterized the interaction among the constituents of a variety of complex systems, ranging from biological to technological, and social systems. However, up until recently, attention was almost exclusively given to networks in which all components were treated on equivalent footing, while neglecting all the extra information about the temporal- or context-related properties

arXiv:1407.0742

155 pages, excellent against insomnia



What about multi-layer functional networks?

Is the multi-layer structure of a functional network relevant?

Can we disregard layers?

Would the resulting dynamics be similar?

Data set available

ALL-FT+

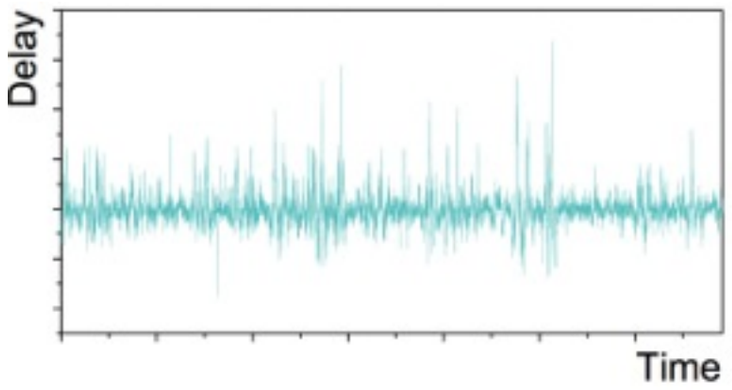
as provided by the PRISME group of EUROCONTROL

All flights crossing the European airspace

1st March - 31st December 2011

> 10 million flights

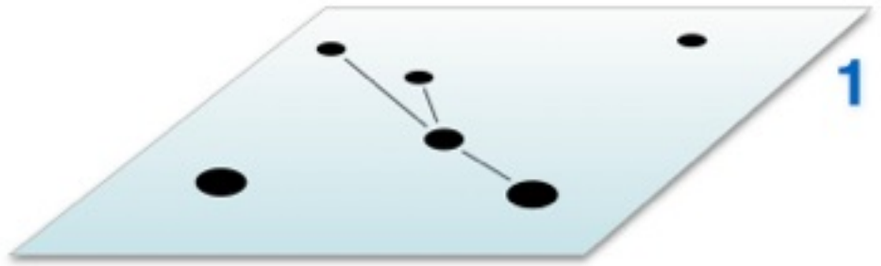
> 400 GB of data

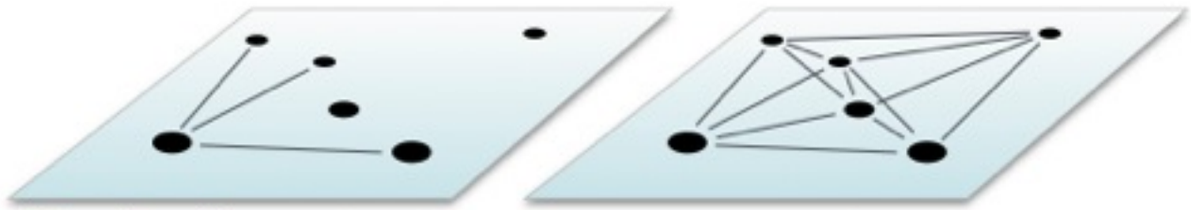


Airport delay time series

LC
MI

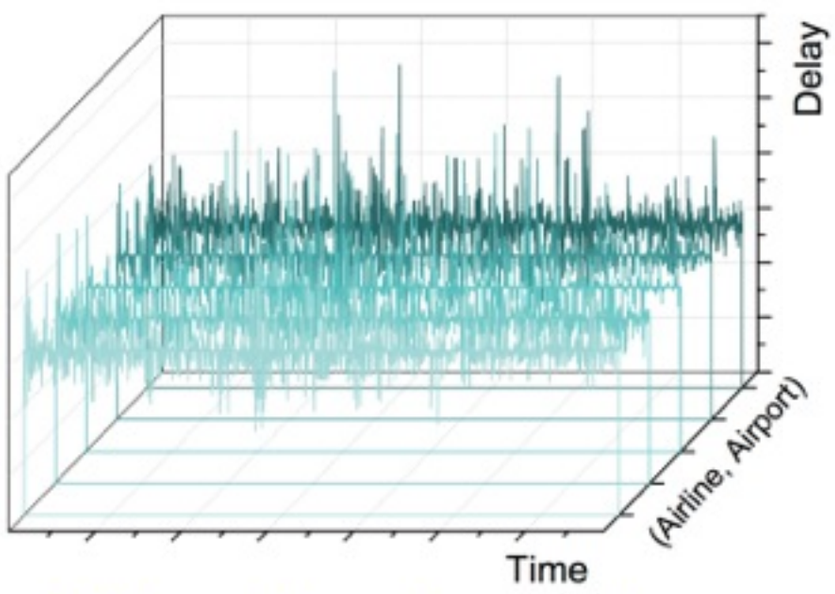
GC
TE





Projection of topology

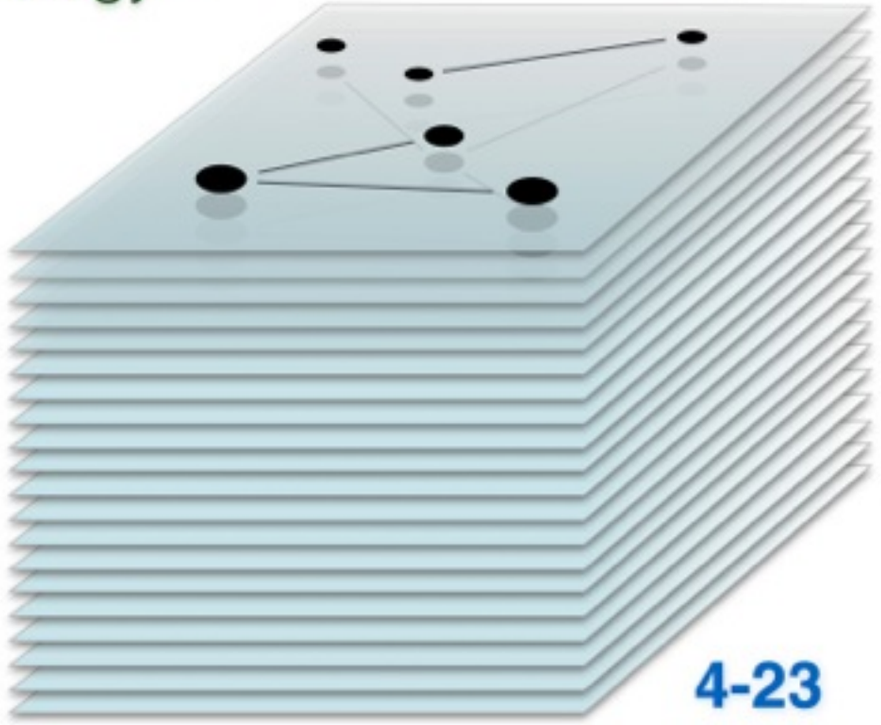
Projection of dynamics



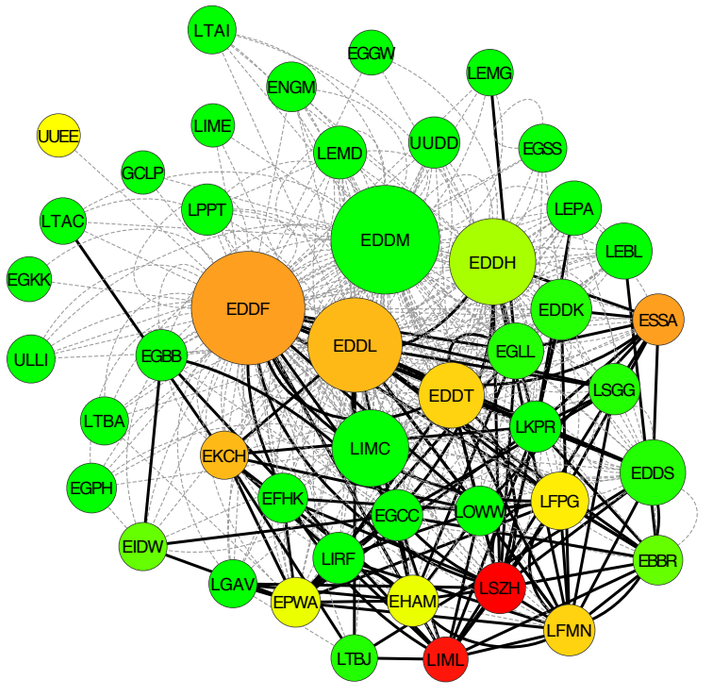
Airline delay time series

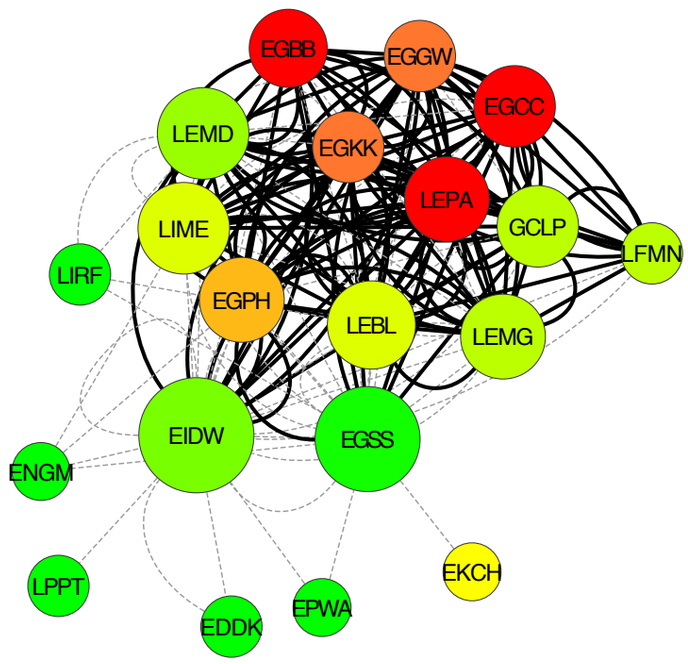
LC
MI

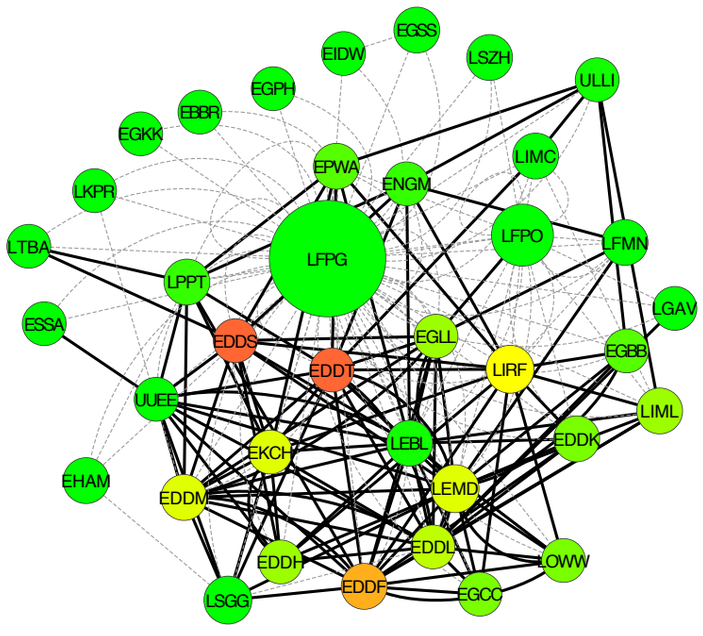
GC
TE

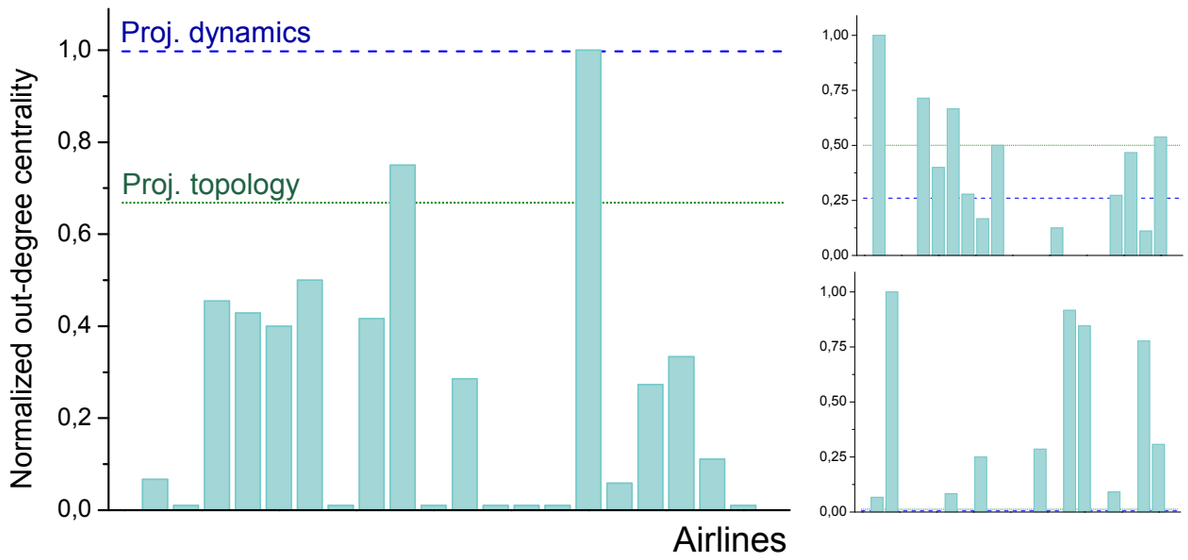


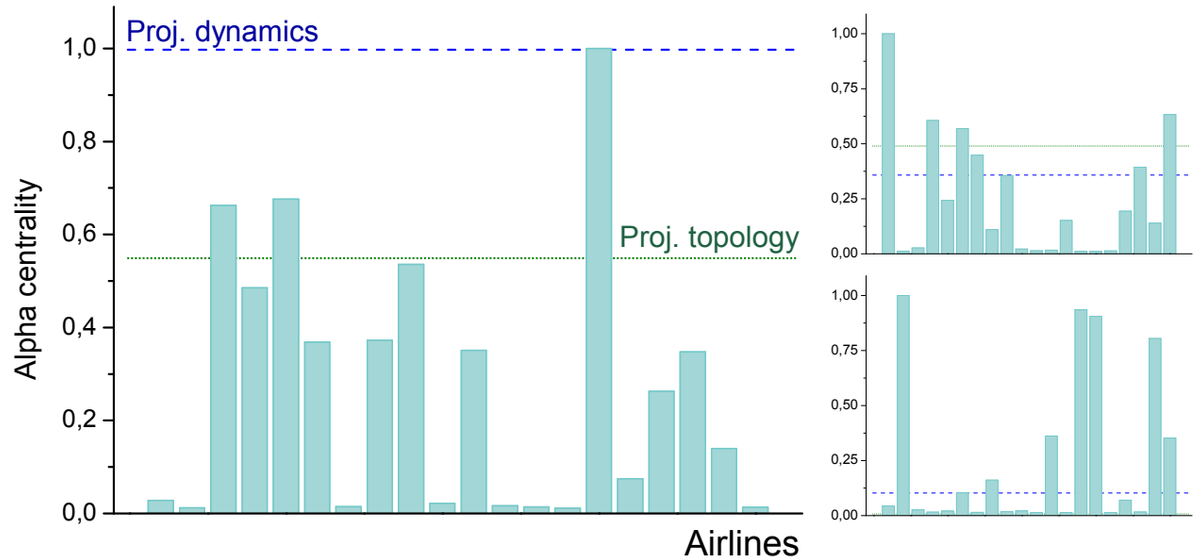
	Projection of dynamics (1)	Projection of topology (2)	Projection of topology (3)	Layer 1	Layer 2	Layer 3
Link density	0.05	0.05	0.54	0.05	0.05	0.05
Maximum out degree	26	16	48	13	13	15
Degree-degree correlation	-0.058	-0.077	-0.029	0.031	-0.088	0.094
Clustering Coefficient	0.022	0.179	0.707	0.064	0.217	0.095
Efficiency	0.064	0.094	0.765	0.110	0.064	0.113
Size giant component	2	17	48	16	14	18

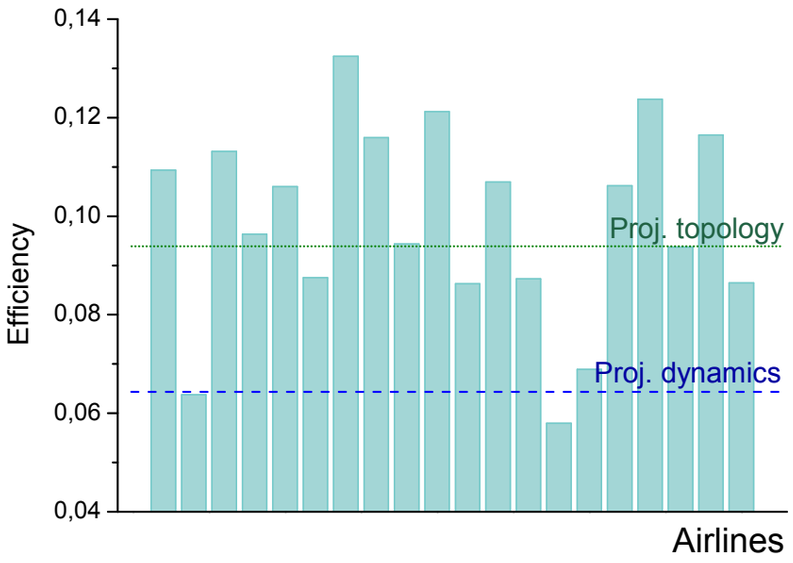


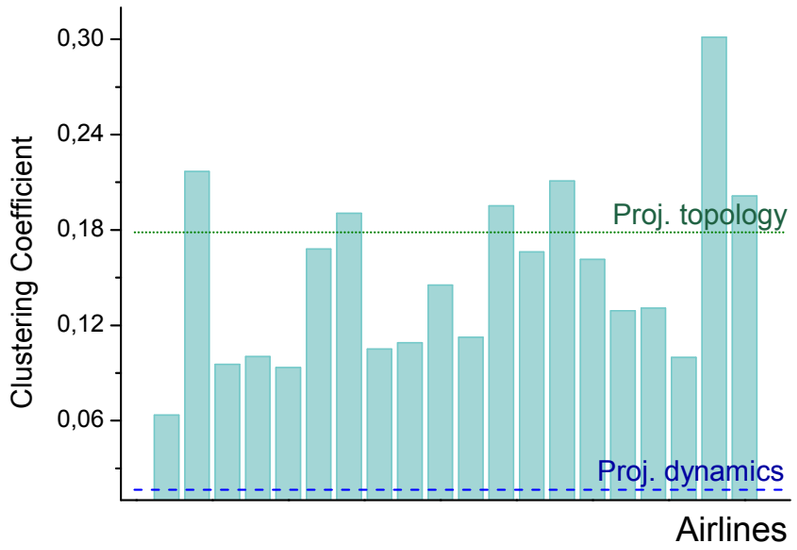


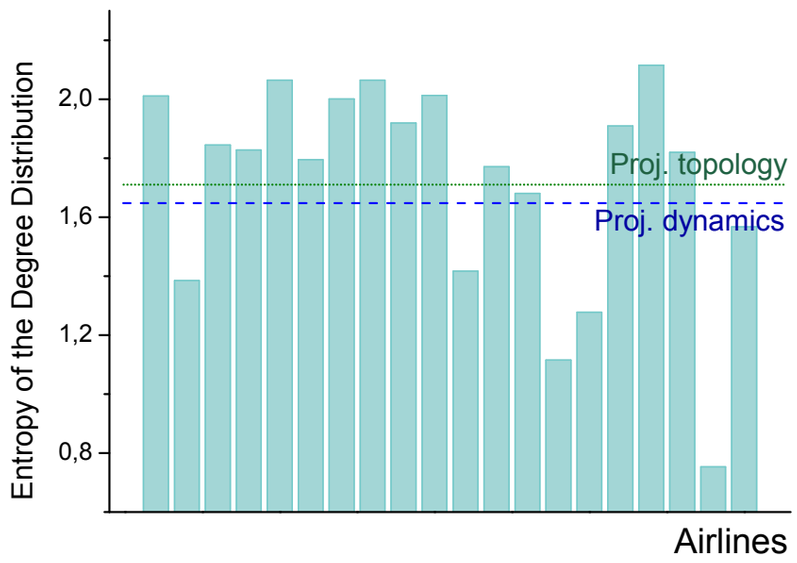



Proj. dynamics
Proj. topology
Normalized out-degree centrality
1,0
0,8
0,6
0,4
0,2
0,0
Airlines
1,00
0,75
0,50
0,25
0,00

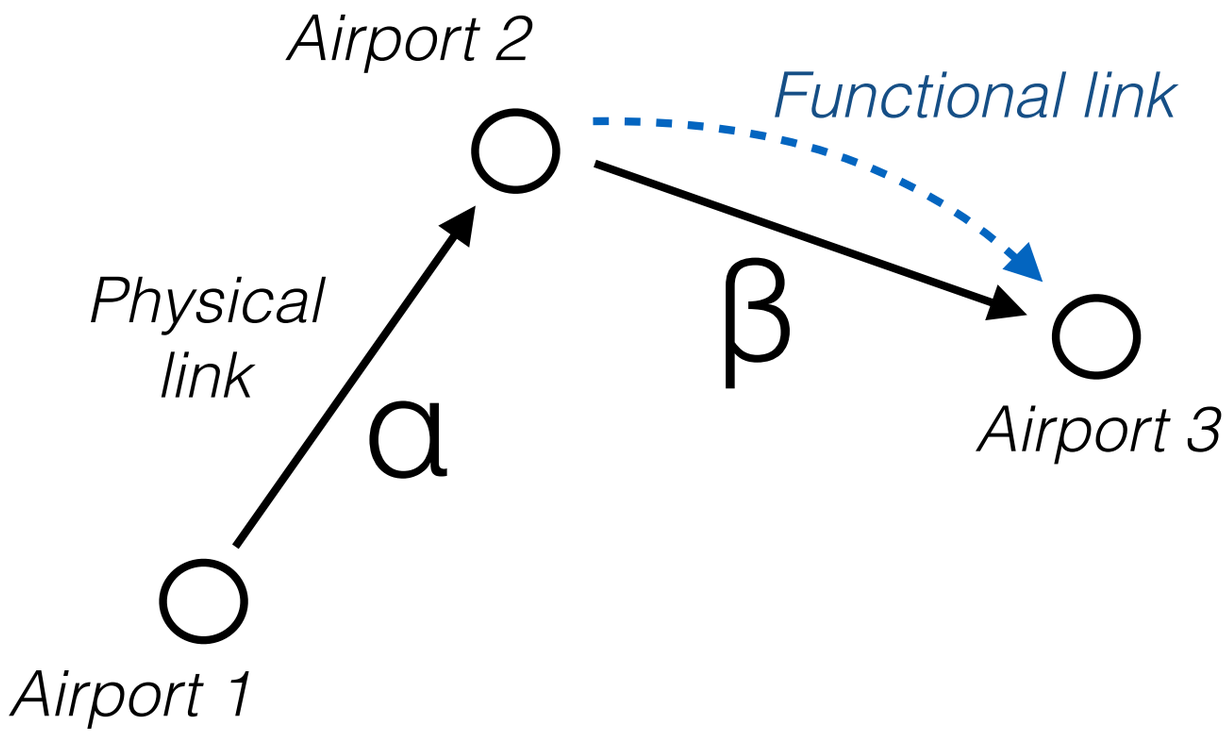

Proj. dynamics
Proj. topology
Alpha centrality
1,0
0,8
0,6
0,4
0,2
0,0
Airlines
1,00
0,75
0,50
0,25
0,00


Proj. topology
Proj. dynamics
Efficiency
0,14
0,12
0,10
0,08
0,06
0,04
Airlines


Proj. topology
Proj. dynamics
Clustering Coefficient
0,30
0,24
0,18
0,12
0,06
Airlines


Proj. topology
Proj. dynamics
Entropy of the Degree Distribution
2,0
1,6
1,2
0,8
Airlines

Dynamical model of delay propagation



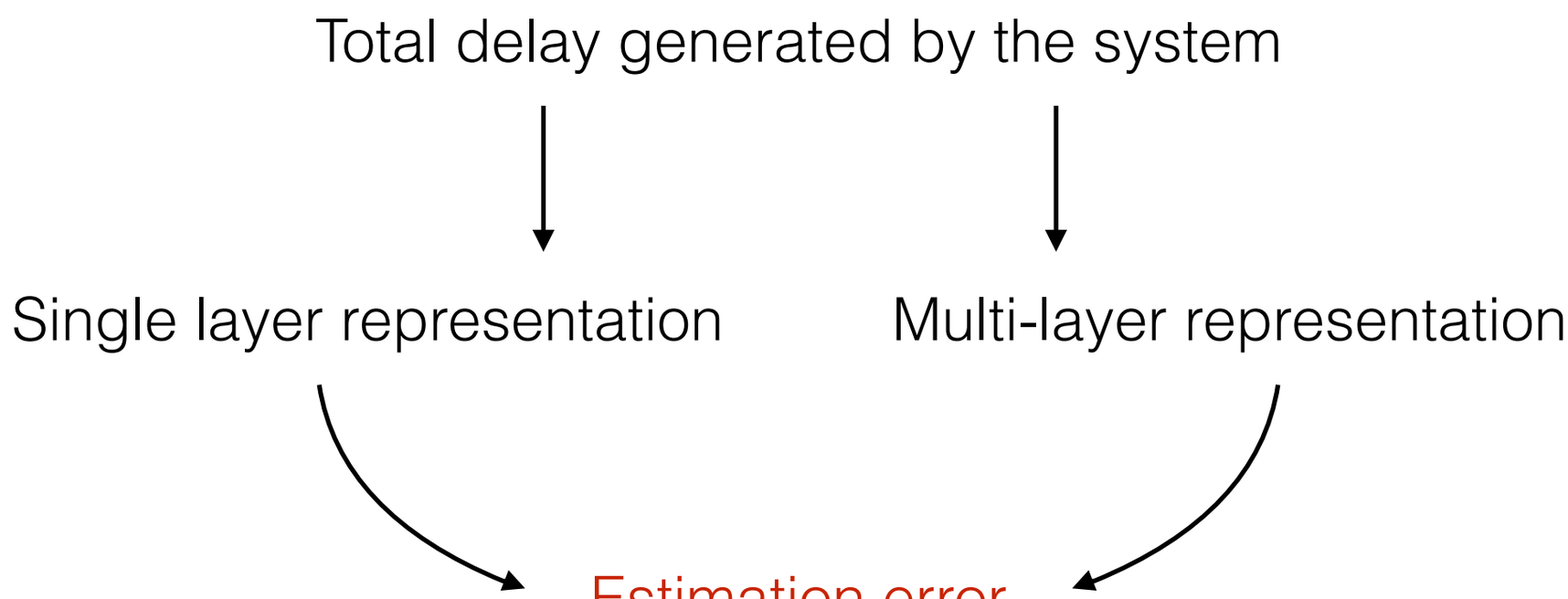
Random walk on the physical network

Delay of each agent defined as: (α, β) * Previous delay
Random term
Negative delays are eliminated

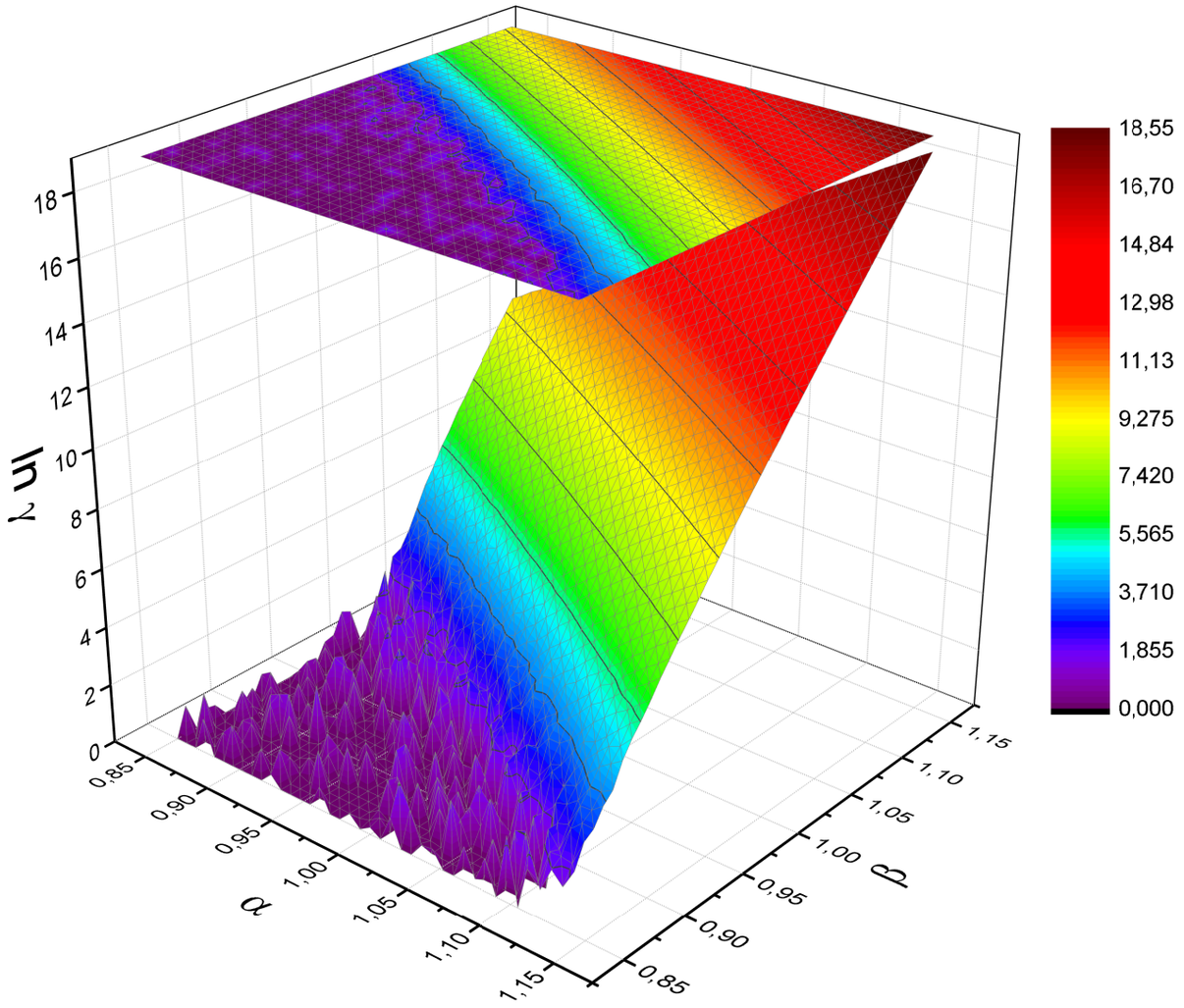
Dynamical model of delay propagation

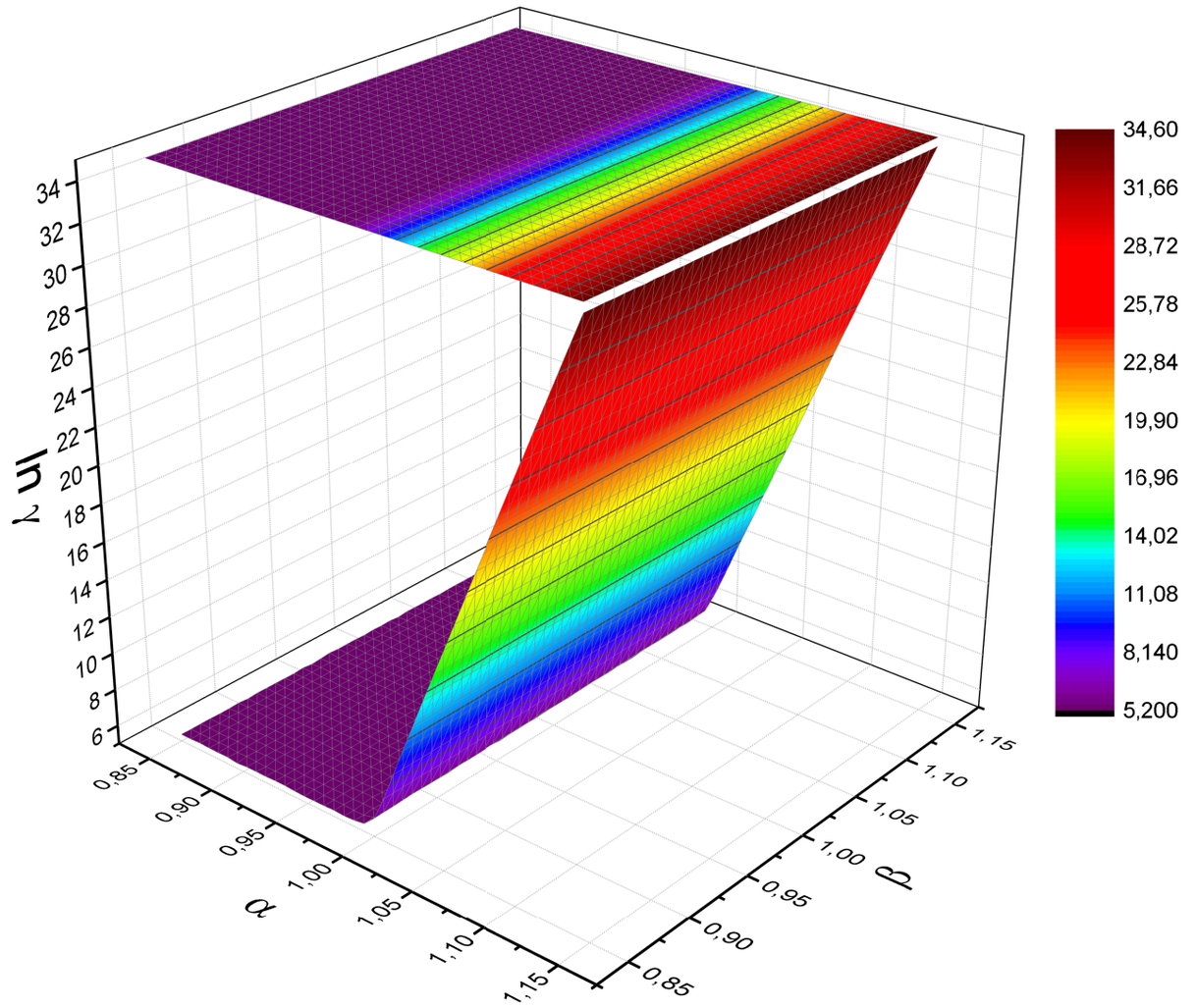
Random walk on the physical network

Delay of each agent defined as: (α, β) * Previous delay
Random term
Negative delays are deleted

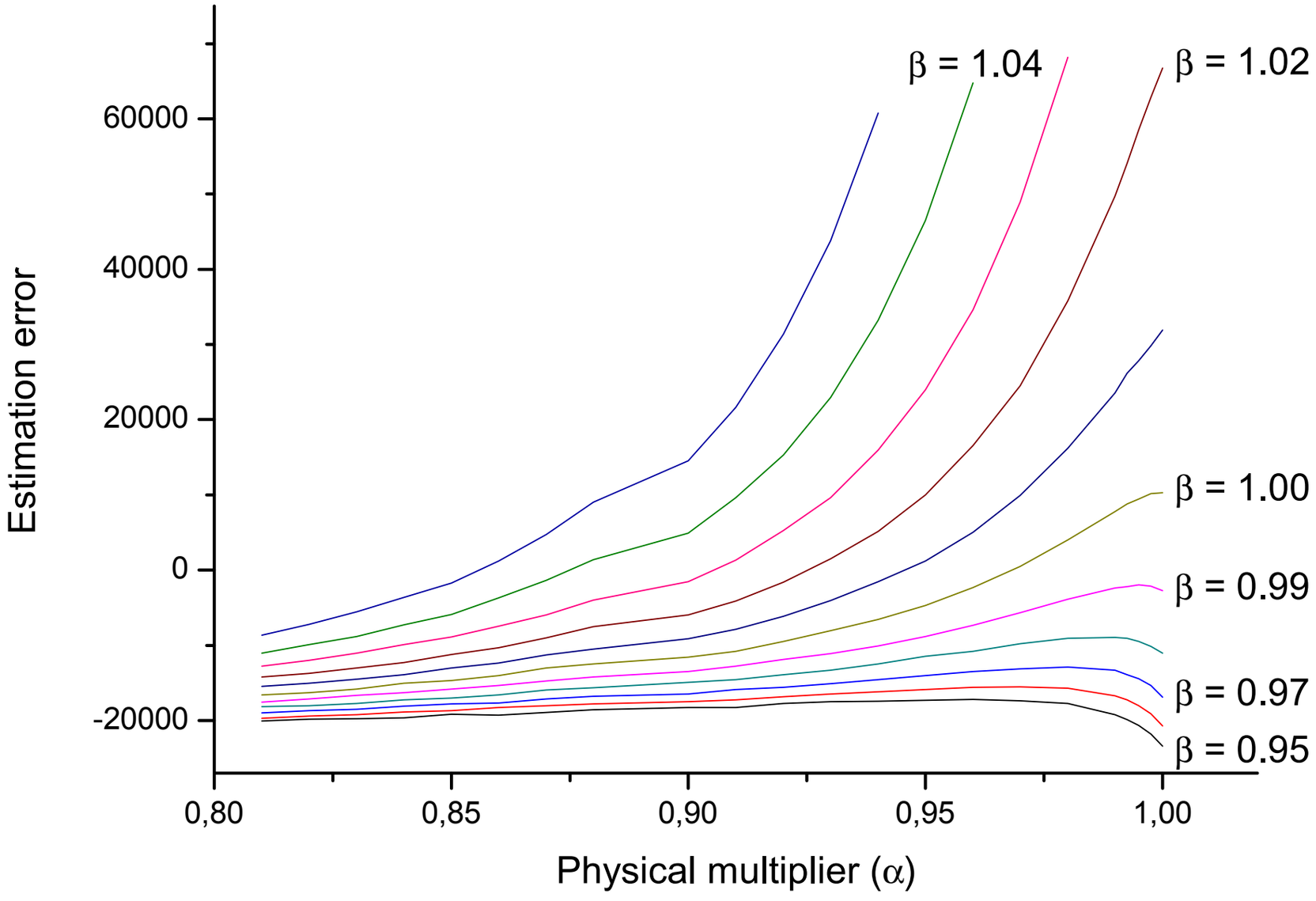


Comparing single *vs.* multi-layer dynamics





Comparing single *vs.* multi-layer dynamics

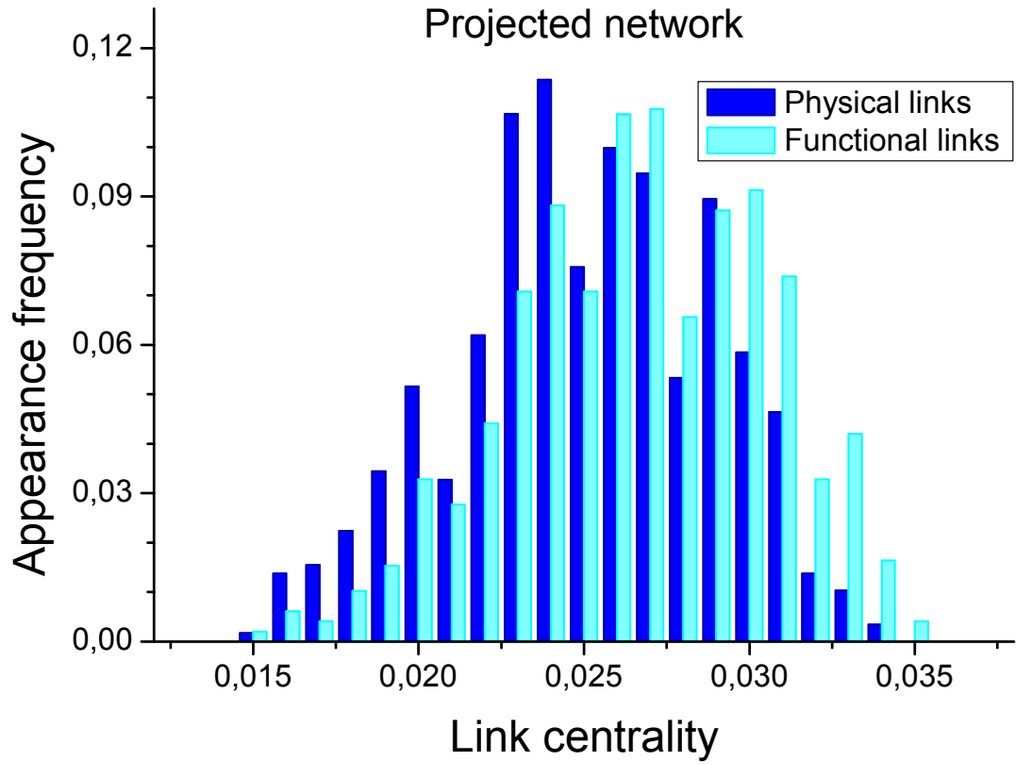


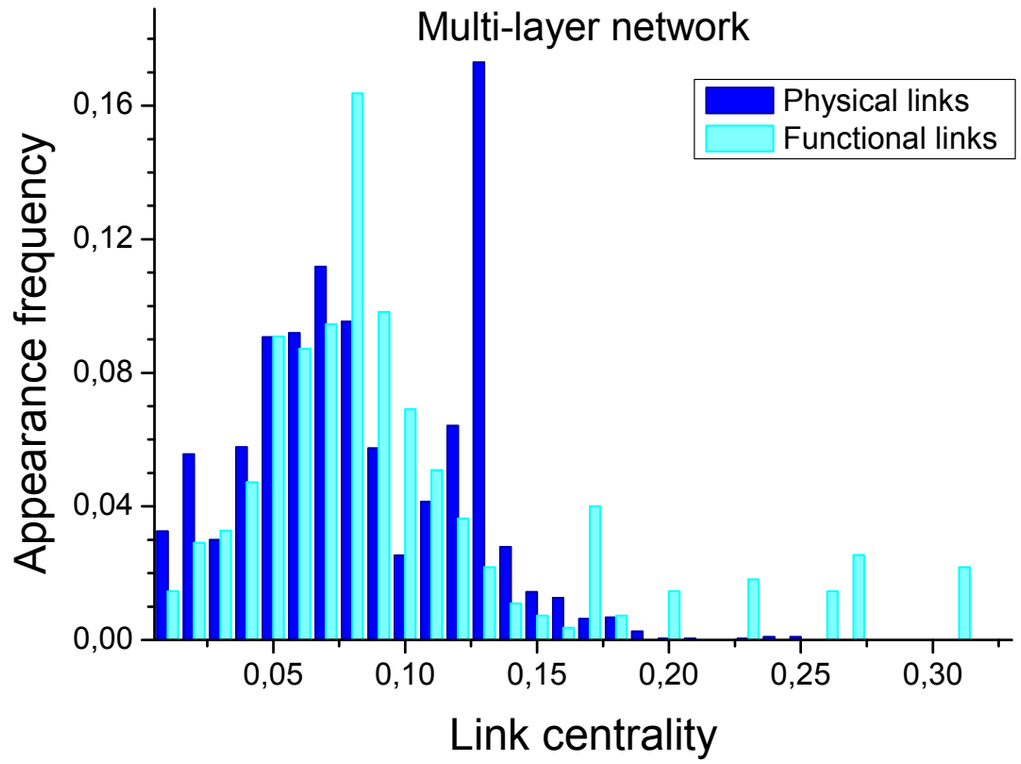
Error of the same order of magnitude than the observable (total delay)

Why such difference?

Distribution of links importance

calculated as α-centrality over the row-normalised adjacency matrix





The importance of key functional links is lost in the projection

Have we got the basics right?

Disregarding the multi-layer structure results in an

erroneous assessment of the structure and dynamics of the system

Wrong understanding of delay propagation

Other *layers* to be considered: aircraft, crews, *etc.*

One network does not fit all!

M. Zanin
Can we neglect the multi-layer structure of functional networks?
In preparation

M. Zanin and F. Lillo
Modelling the air transport with complex networks: A short review
Eur. Phys. J. Special Topics 215, 5–21 (2013)

A. Cardillo, J. Gómez-Gardeñes, M. Zanin *et al.*
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Thank you