LASAGNE
Cutting through complexity

Spatio-temporal complex networks: reachability, centrality, and robustness

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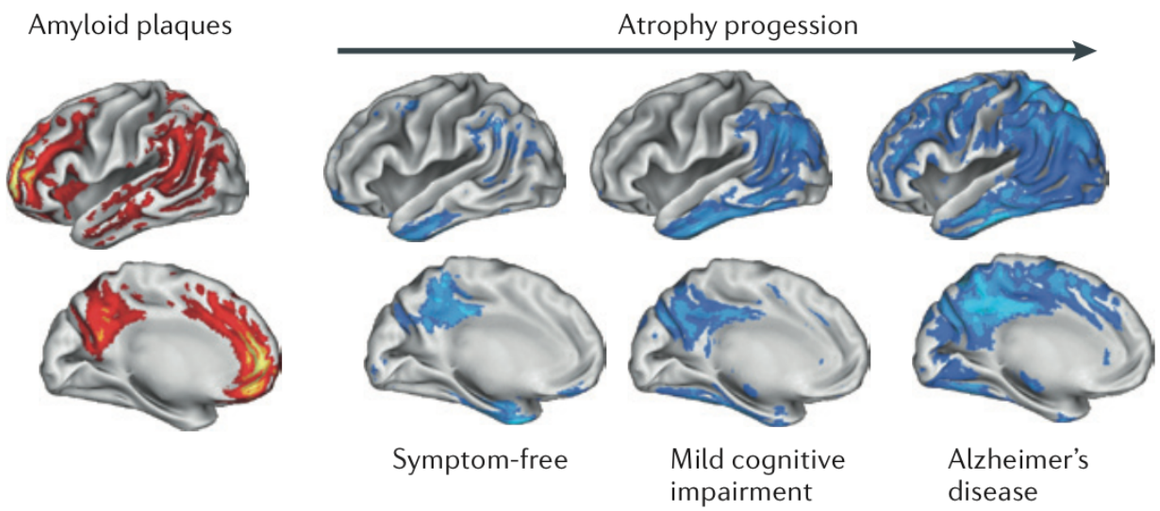
UNIVERSITY OF BIRMINGHAM

UCL

What is the impact of failures in a spatio-temporal system?



[Bullmore & Sporns, 2012]

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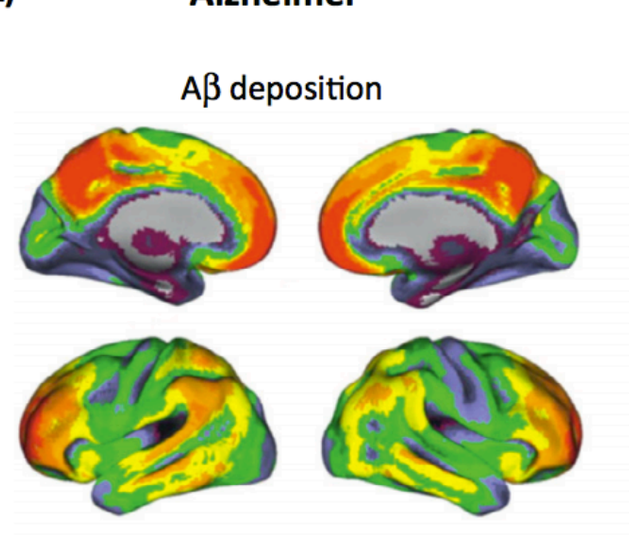
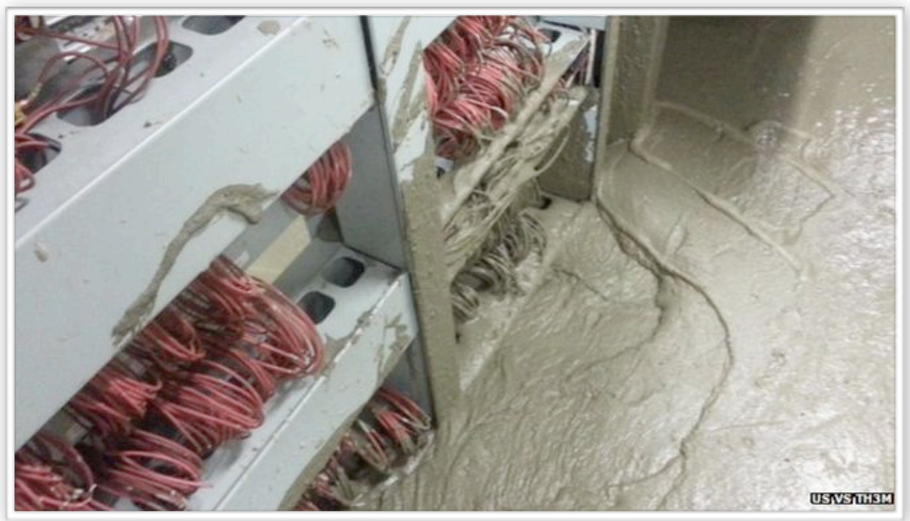


[van den Heuvel & Sporns, 2013]

Victoria Tube line part shut hit by wet concrete flood

23 January 2014 | London

BBC NEWS



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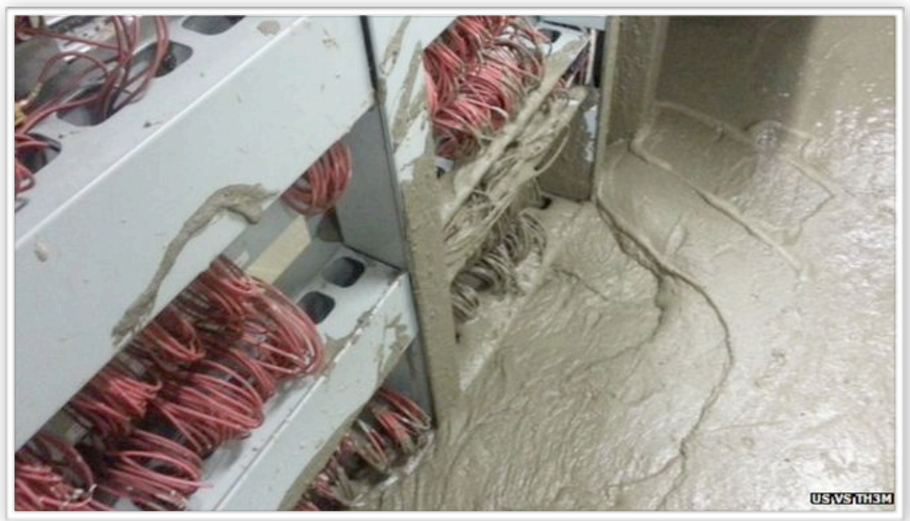


What is the impact of failures in a spatio-temporal system?

Victoria line
@victorialine

Follow

There's no service btn Warren Street and Brixton while we fix damage caused by flooding at Victoria. **Severe delays on the rest of the line.**

3:40 PM - 23 Jan 2014

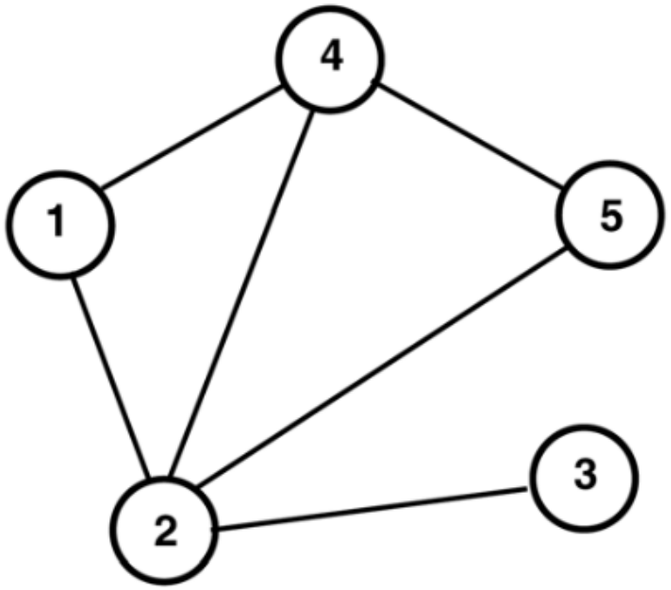
Outline

- Quick intro to (time-binned) temporal networks
- Features of **spatio-temporal networks**
- **Spatio-temporal paths** over networks
- Measuring the **performance** of spatio-temporal networks
- **Robustness** to random failure and systematic attack in **real-world networks**

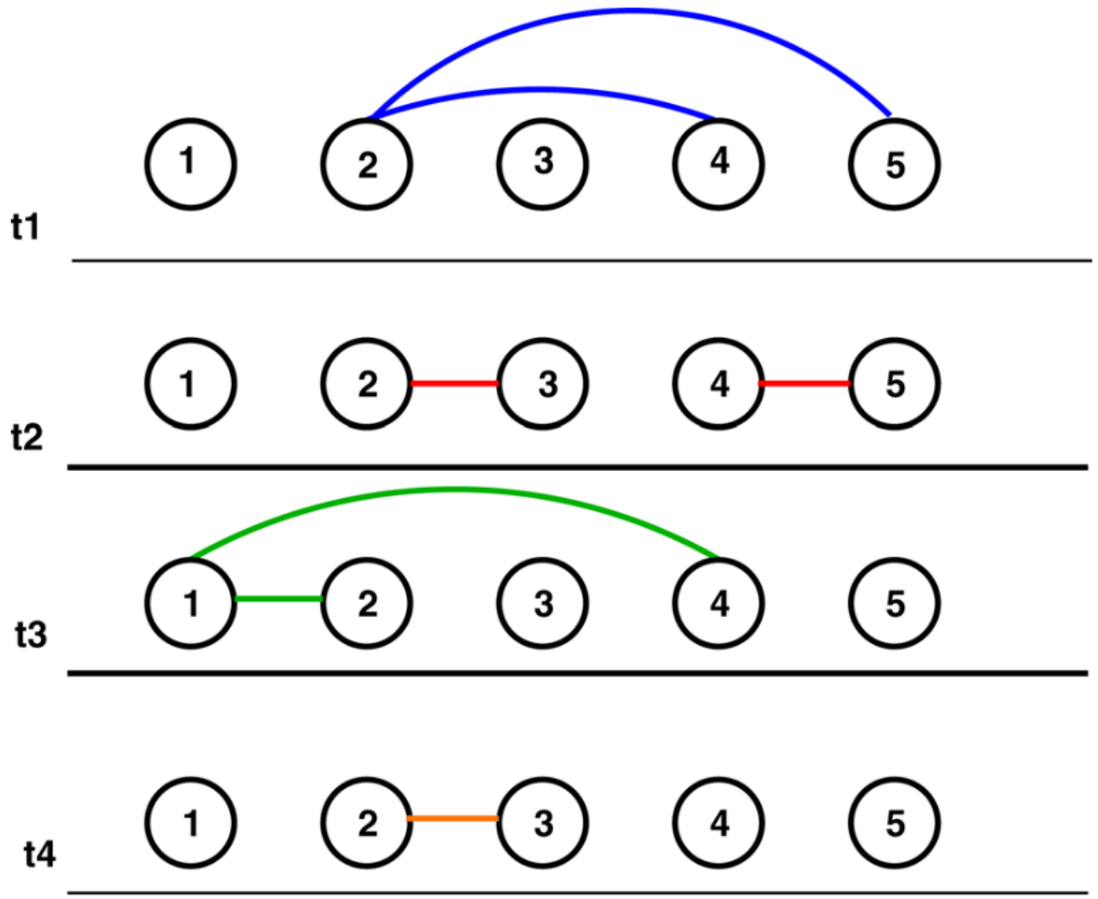
Why Temporal Networks?

- Many networks are **time-evolving**. How do we understand the network in terms of its time-evolving connectivity?
- **Toolbox**: Time-binned representation, reachability, time-respecting paths ("temporal paths"), temporal components, +more
- **Early applications**: Mobile opportunistic networks, wildlife sensor networks, mobile malware defence, e-mail networks
- **Recent applications**: Transport networks, infrastructure systems, social media information dissemination

(Time-Binned) Temporal Networks

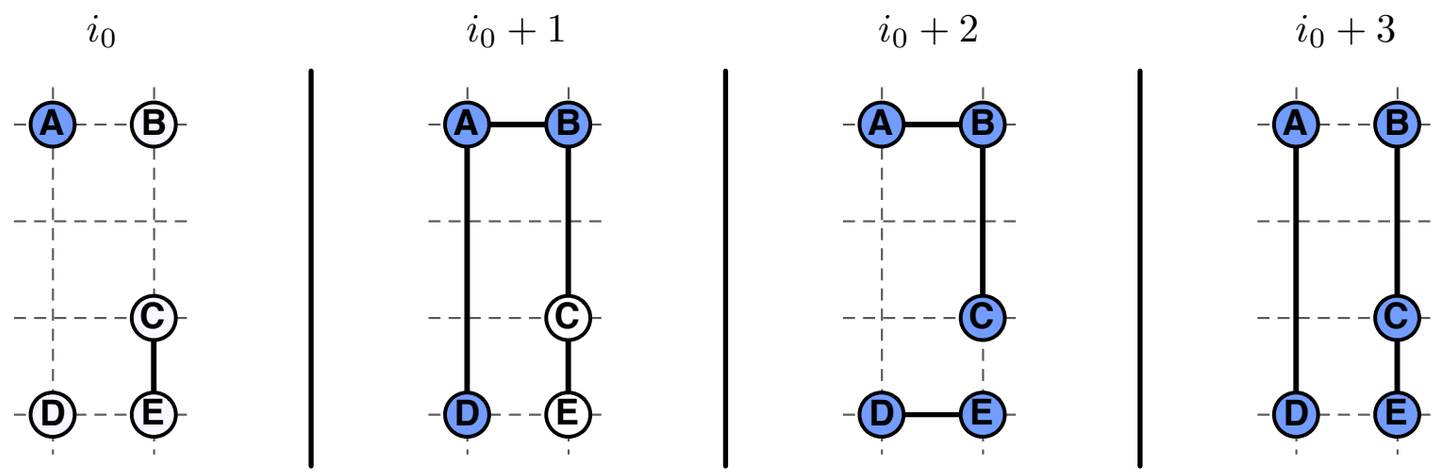


Aggregate (Static) Network



Underlying Temporal Network

Reachability

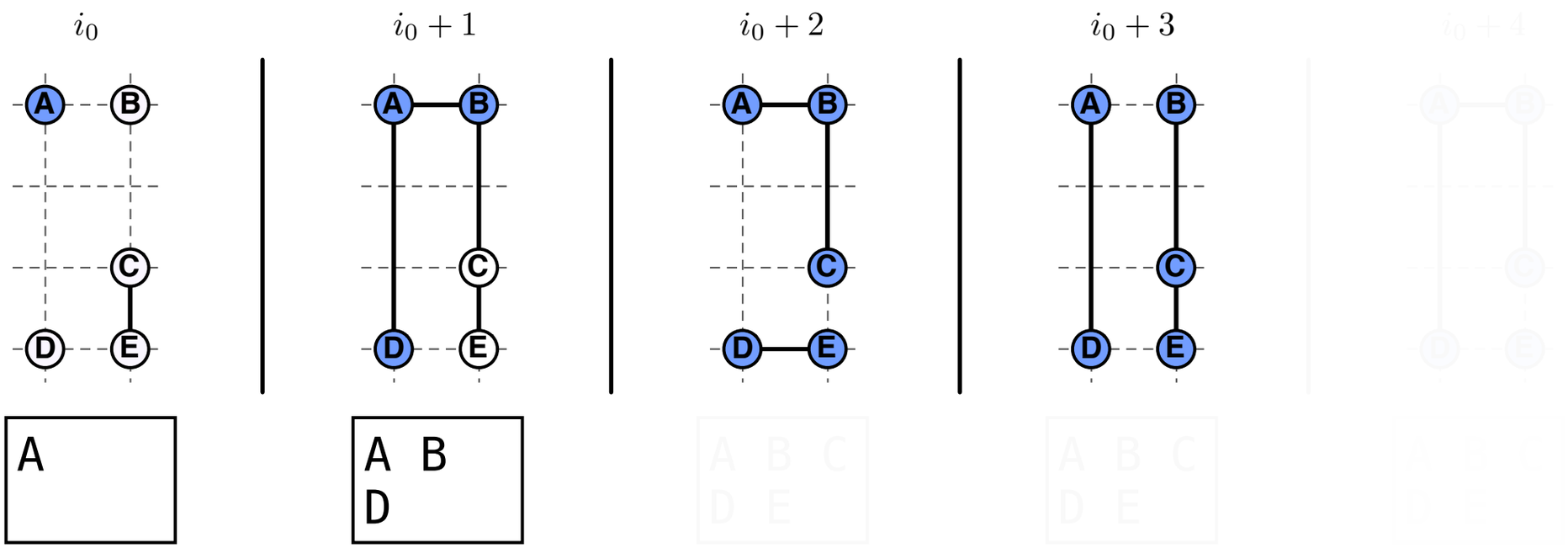


A

Consider propagation from A...

- What other nodes can we reach?
- When do we reach them?

Reachability



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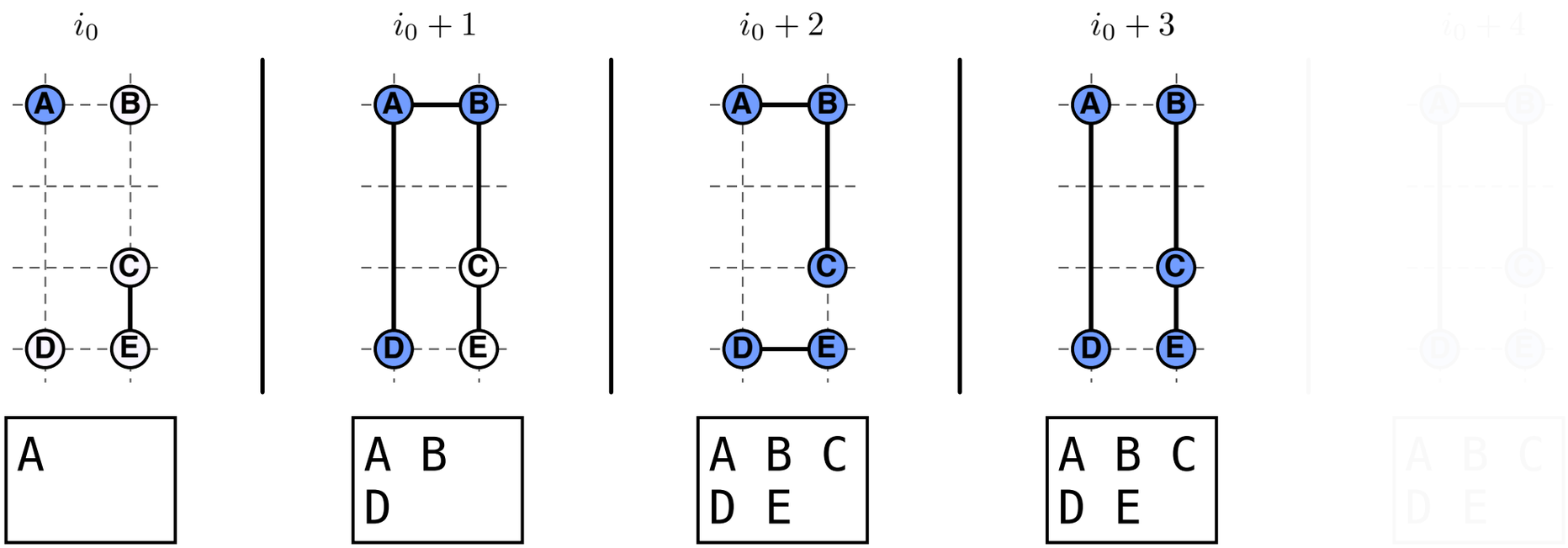

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Temporal Paths

Propagation model → **Temporal paths**
Sequence of successful node-to-node propagation events

Temporal path:

$$(v_0, t_1), (v_1, t'_{\mathrm{arr}_1}), (v_2, t'_{\mathrm{arr}_2}), \ldots, (v_n, t'_{\mathrm{arr}_n})$$

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(origin node, start time)

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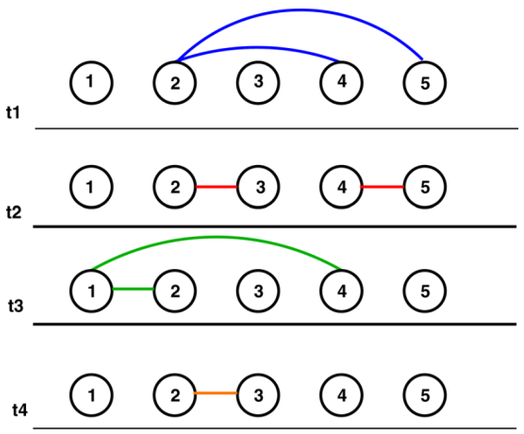
*(**origin node**, **start time**)*

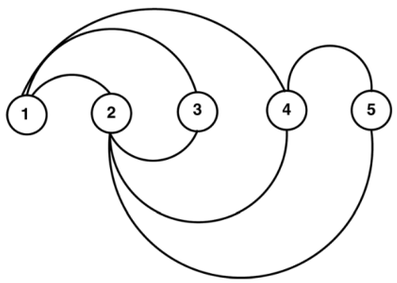
*(**node** v, **time** t) ...*

Giant Temporal Components

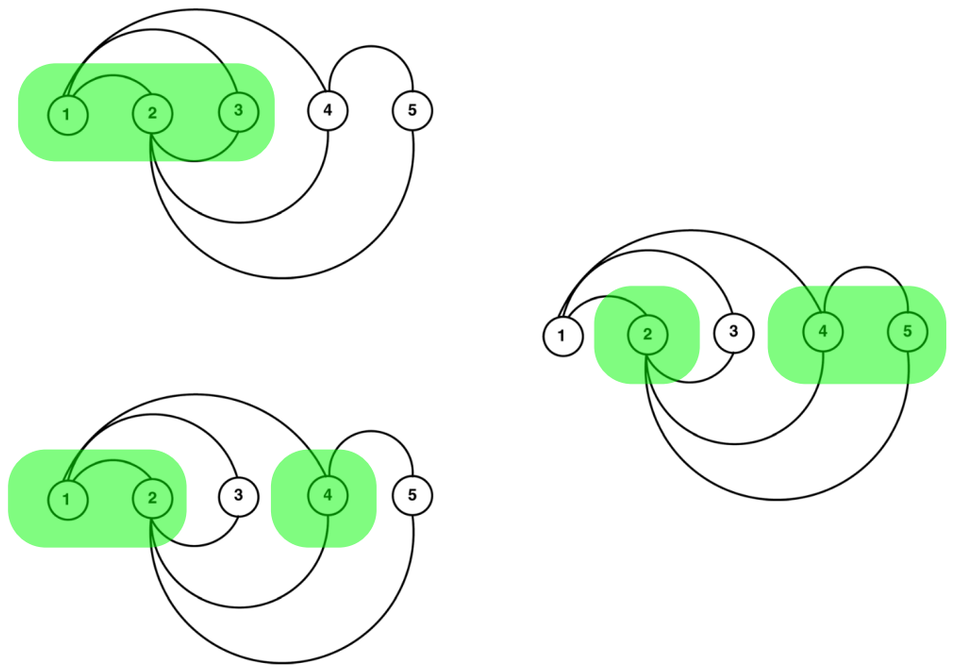
- **Strongly connected temporal component**: Component in which all nodes are **mutually reachable** by a **temporal path**
- Via maximum clique finding – NP-complete



Temporal Network



Affine graph (Mutual reachability)



Maximum clique(s) (Giant temporal component)

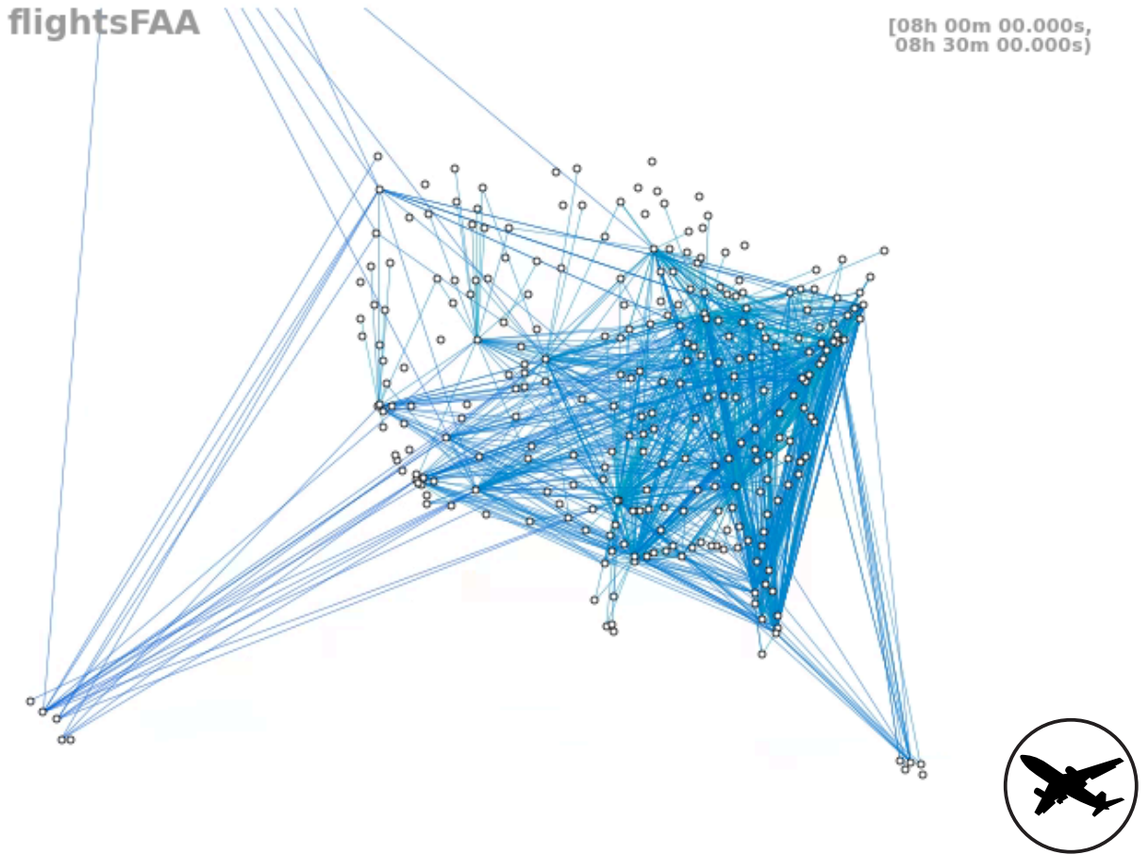
[Nicosia et al, 2011]

Generalised Spatio-Temporal Networks

- **Spatial**:
  Nodes and edges embedded in (metric) space
- **Mobile**:
  Nodes may be mobile (time-varying location)
- **Temporal**:
  Time-evolving topology
- **Non-instantaneous interaction**:
  Node-to-node interactions are *constrained by space* and may be *non-instantaneous*

Example: Public Transport

London Underground
(Metro Rapid Transit System)

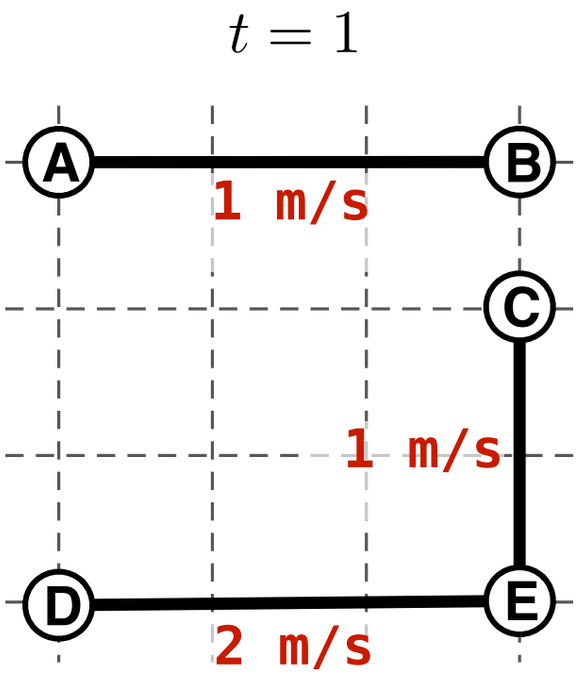


US Domestic Flights

Process over the network = Passenger transit

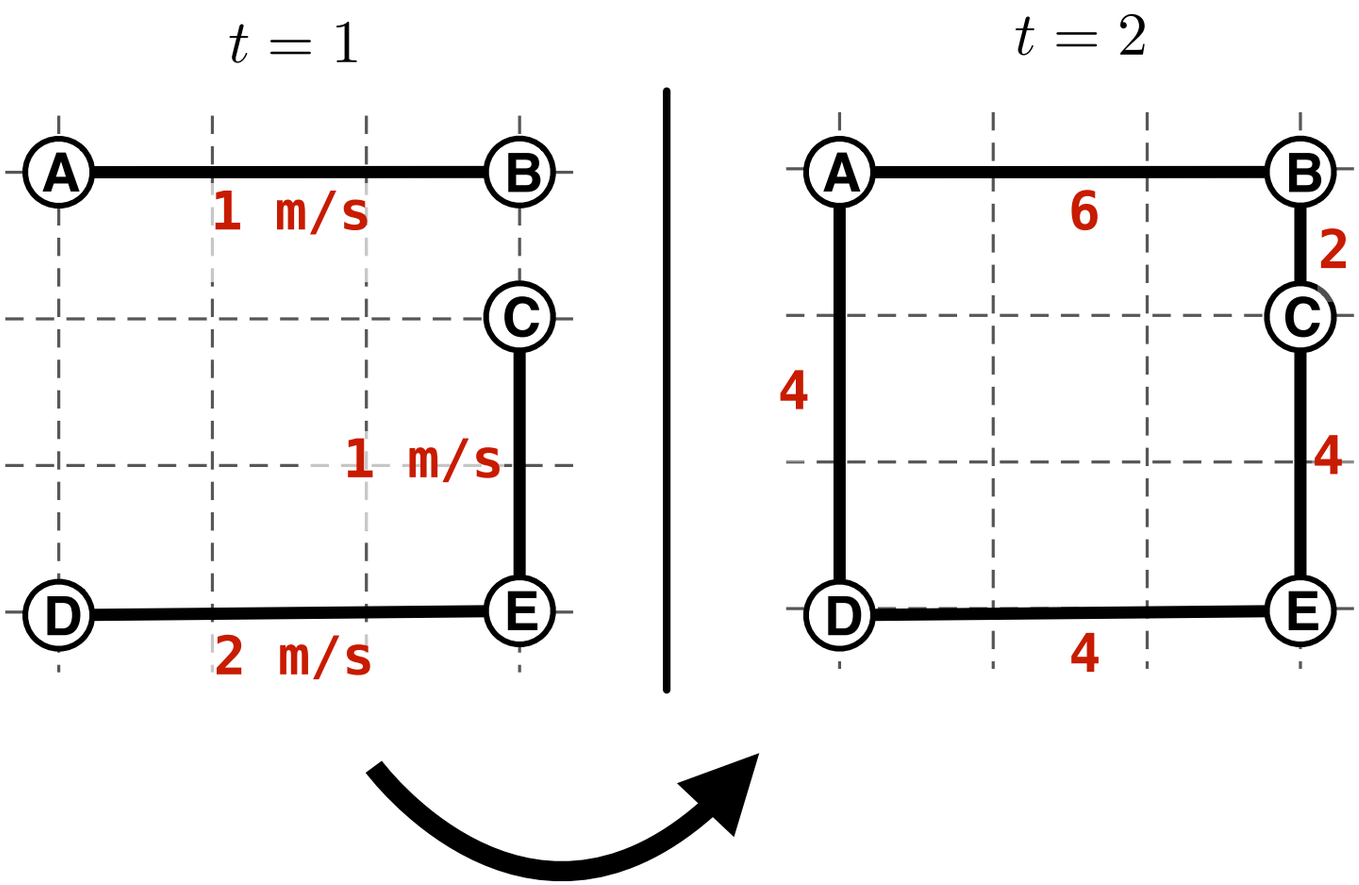
Representation of spatio-temporal networks

Representation



- Time-varying network
- Encode **propagation speed** on each (directed) link
- Possibly infinite for instantaneous transmission networks
- Allows us to derive the **interaction delay** for a pair of nodes

Representation



time-varying...
links &
propagation speeds

(e.g., transit speeds)

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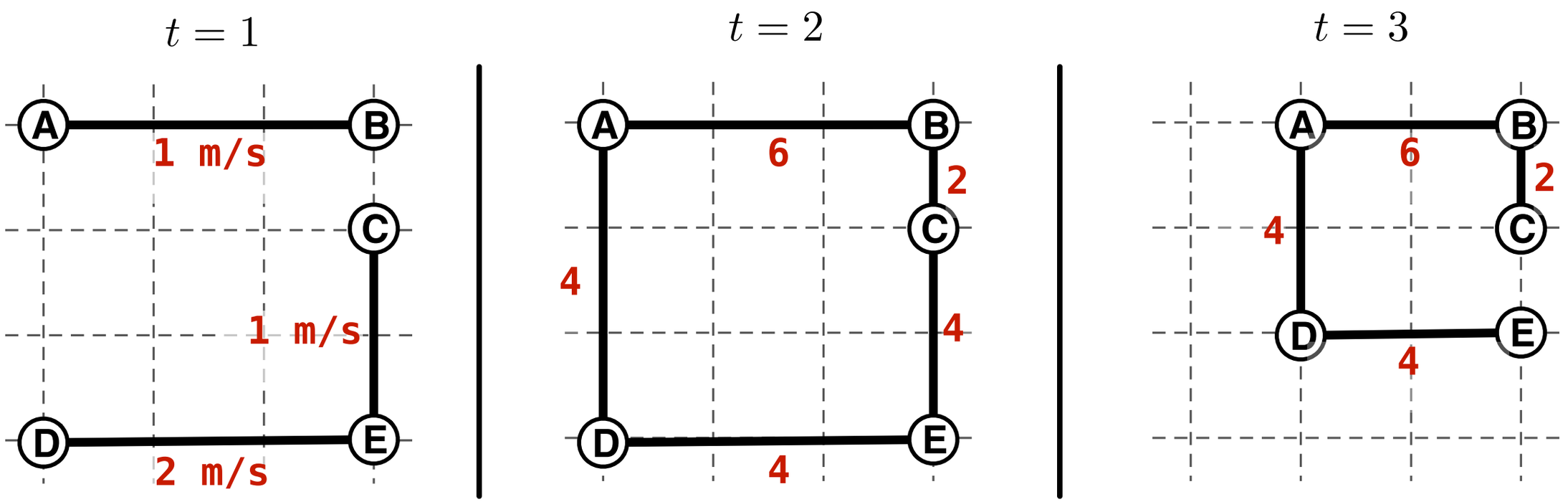


time-varying...
links &
propagation speeds

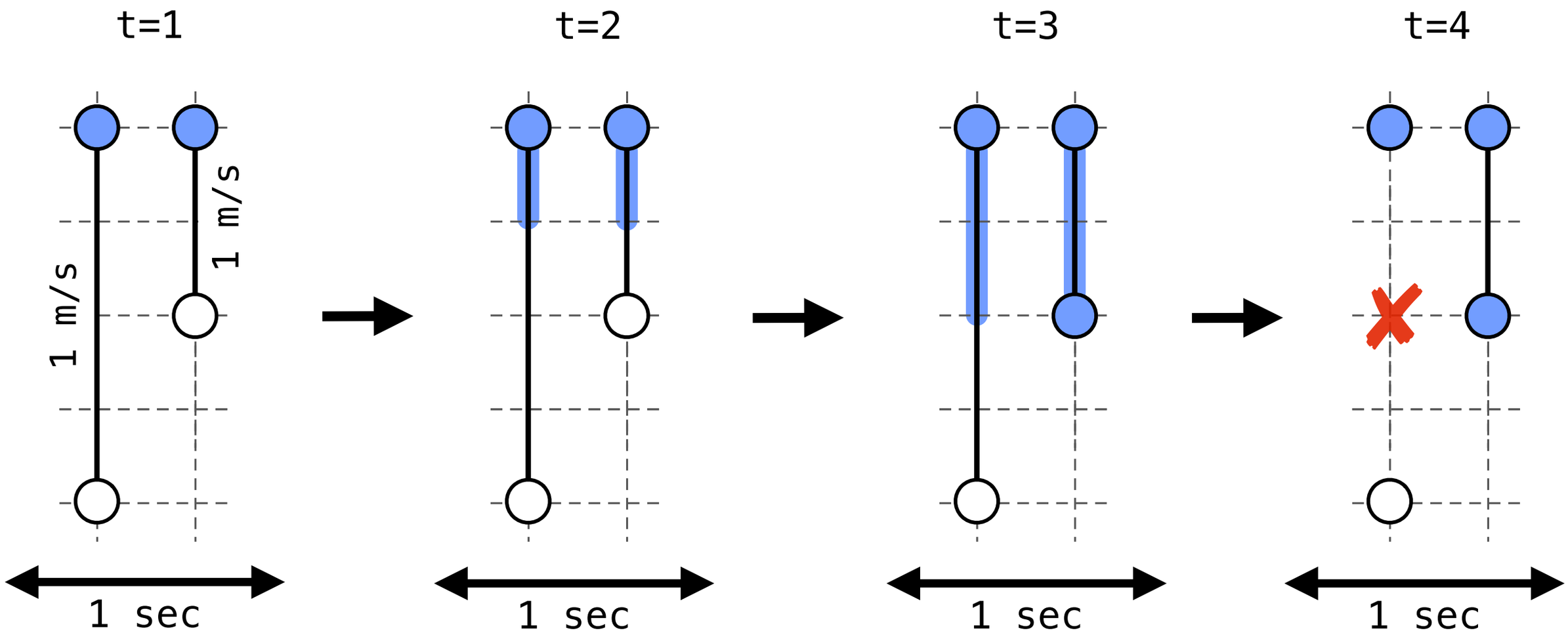
(e.g., transit speeds)

time-varying...
node positions

(e.g., mobile phone comms)

Defining paths over spatio-temporal networks

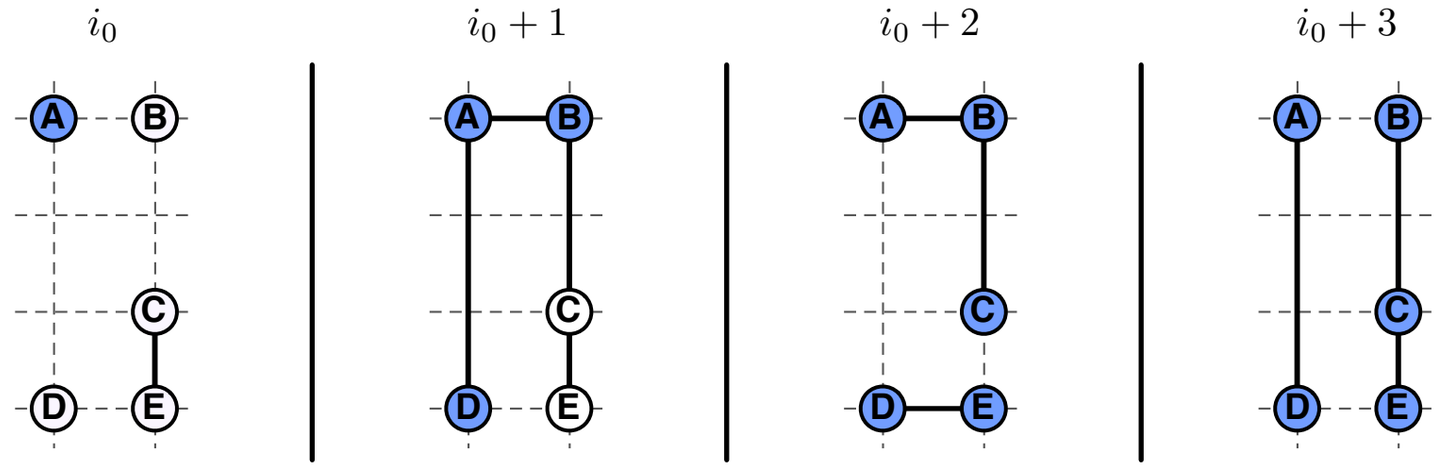
Constrained Propagation: Direct Case



- Model **partial propagation** between nodes at each timestep
- Increment progress between two nodes according to their **physical distance** and the **propagation speed of their link**
- Absence of a link '**resets**' the process between two nodes

temporal network

instantaneous transmission

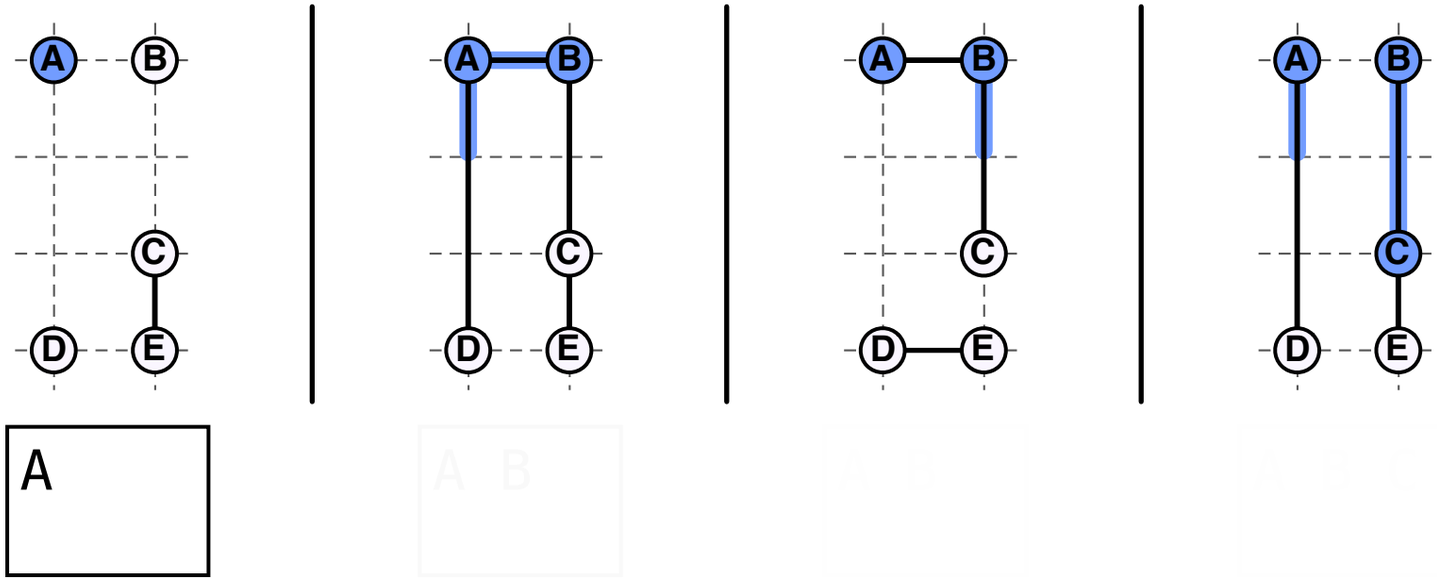


Consider propagation from A...

- What other nodes can we reach?
- When do we reach them?
- What distance did we travel?

spatio-temporal network

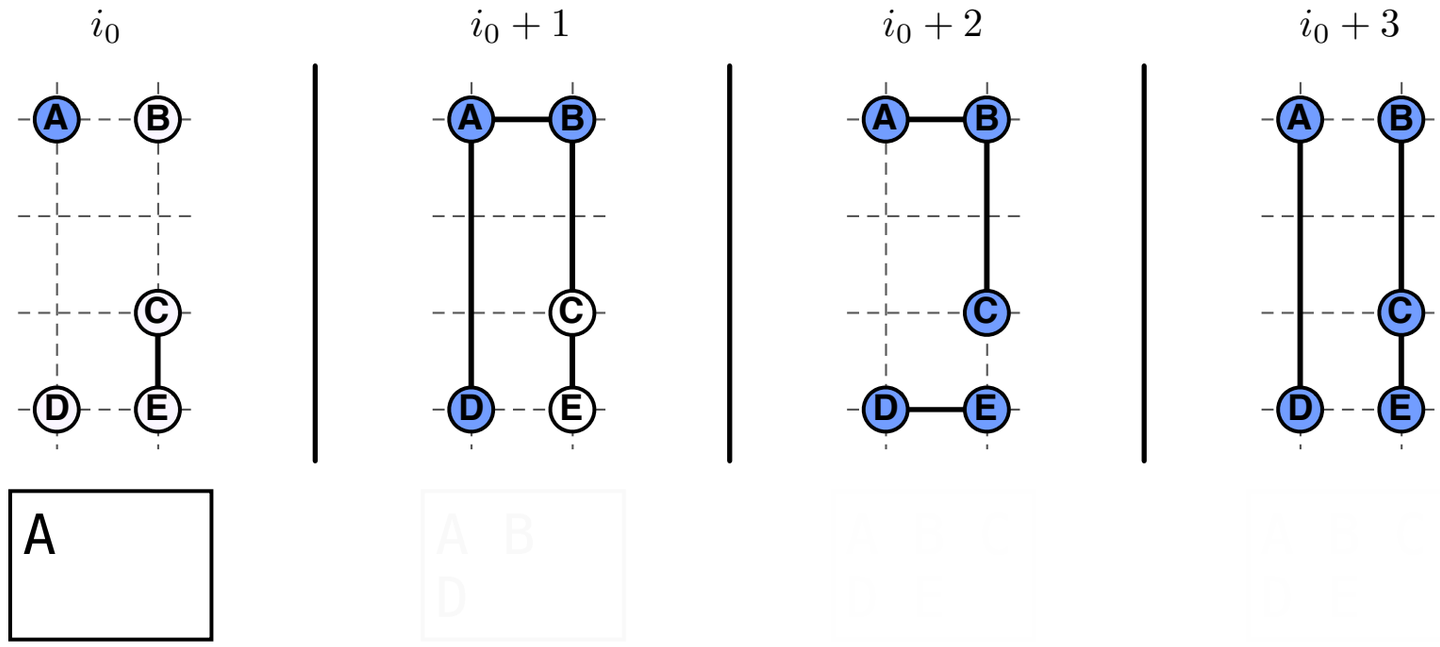
constrained by propagation speed of each link



propagation speeds = 1 metre/timestep *grid = 1x1 metre*

temporal network

instantaneous transmission



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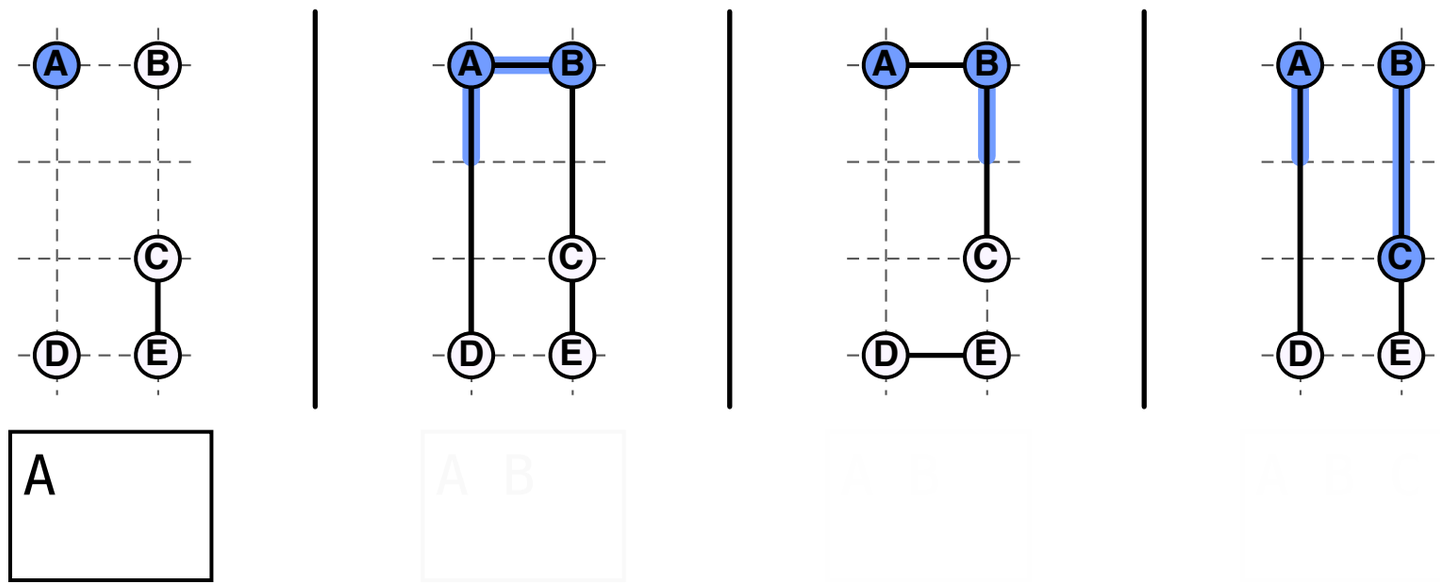

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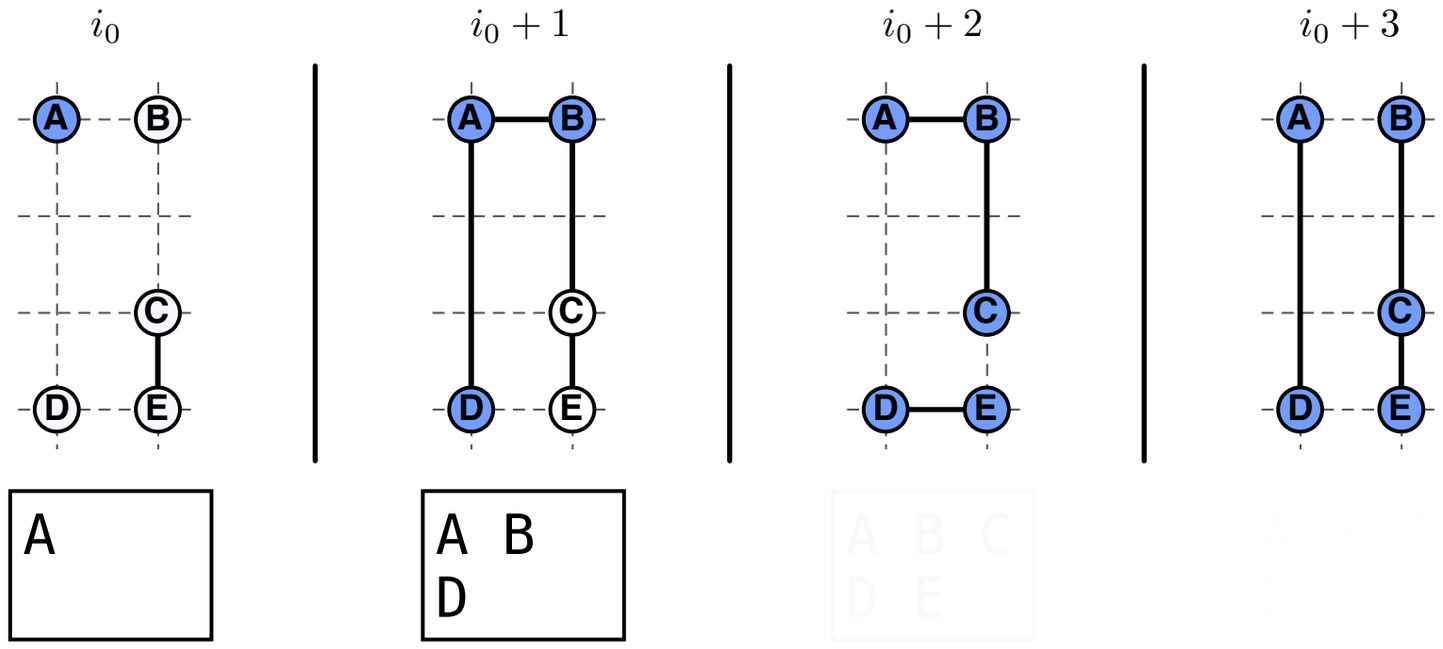

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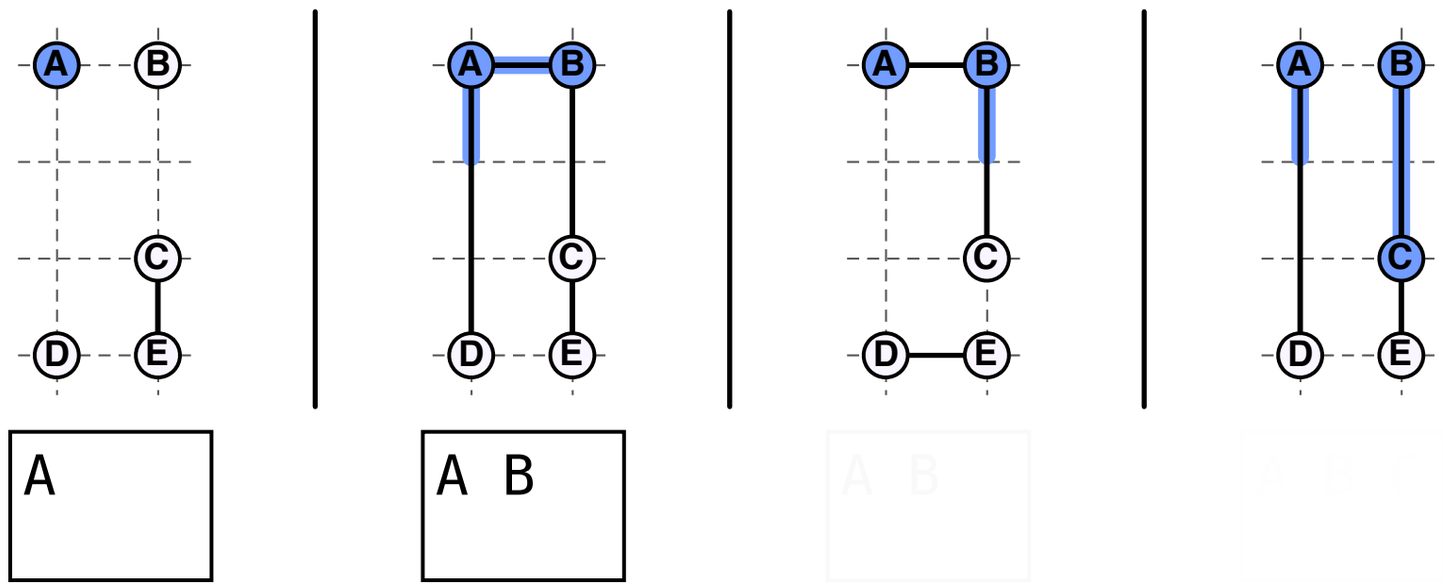

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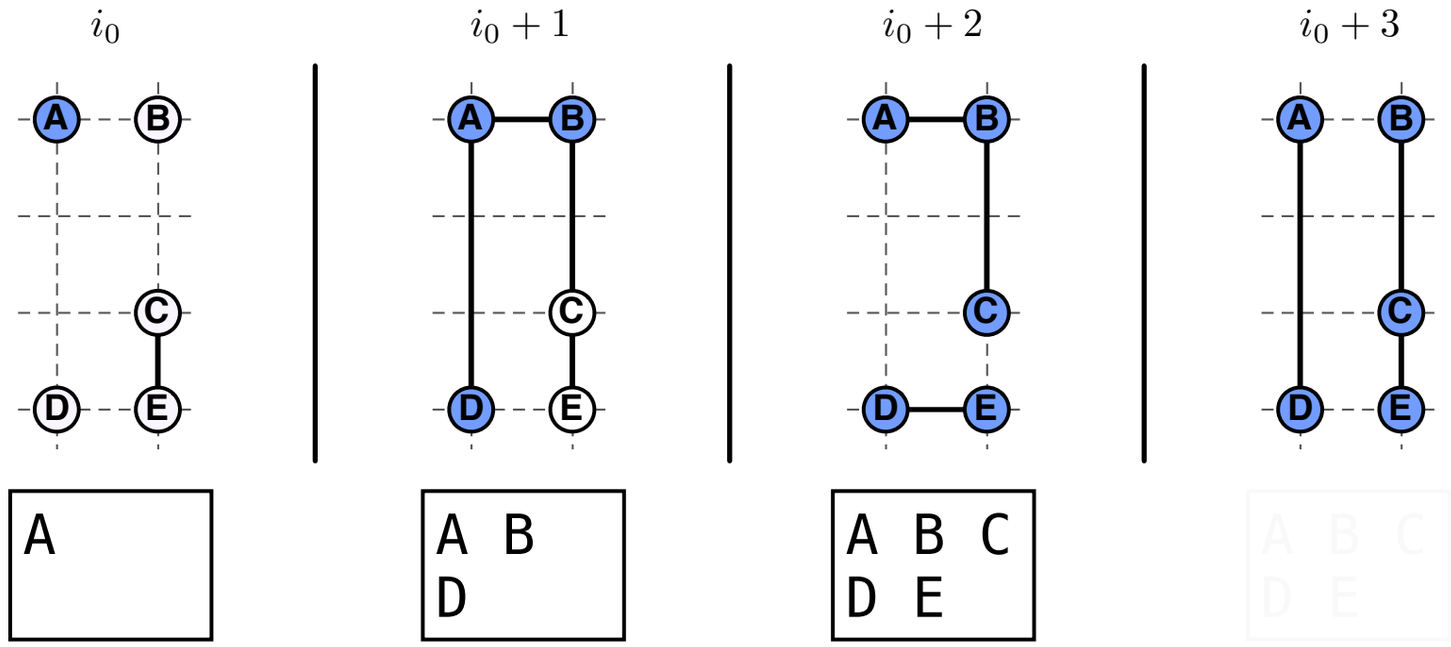

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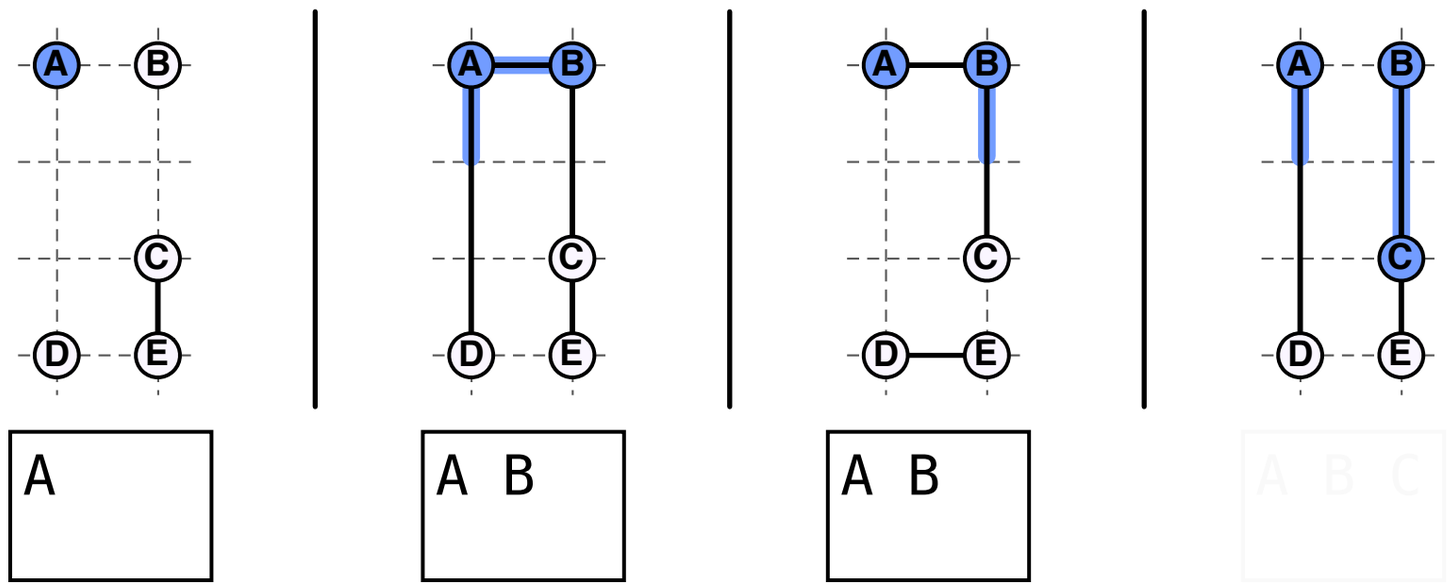

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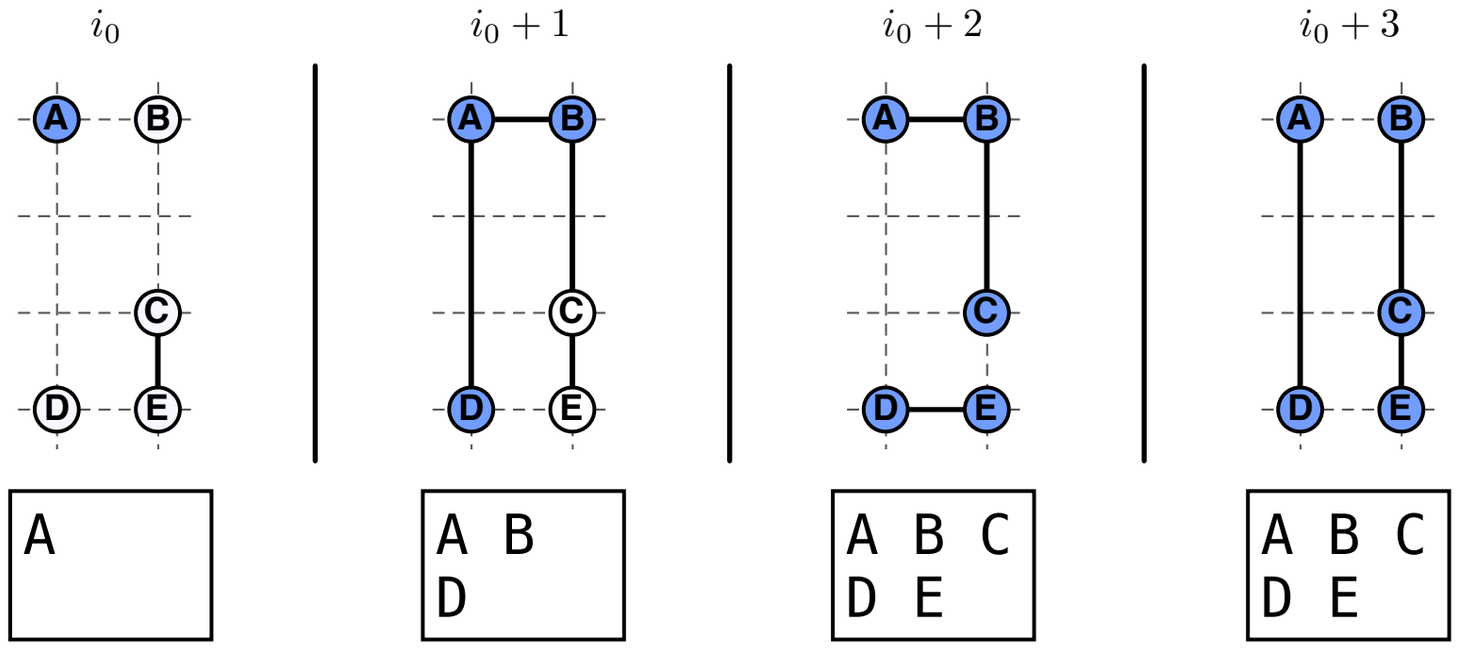

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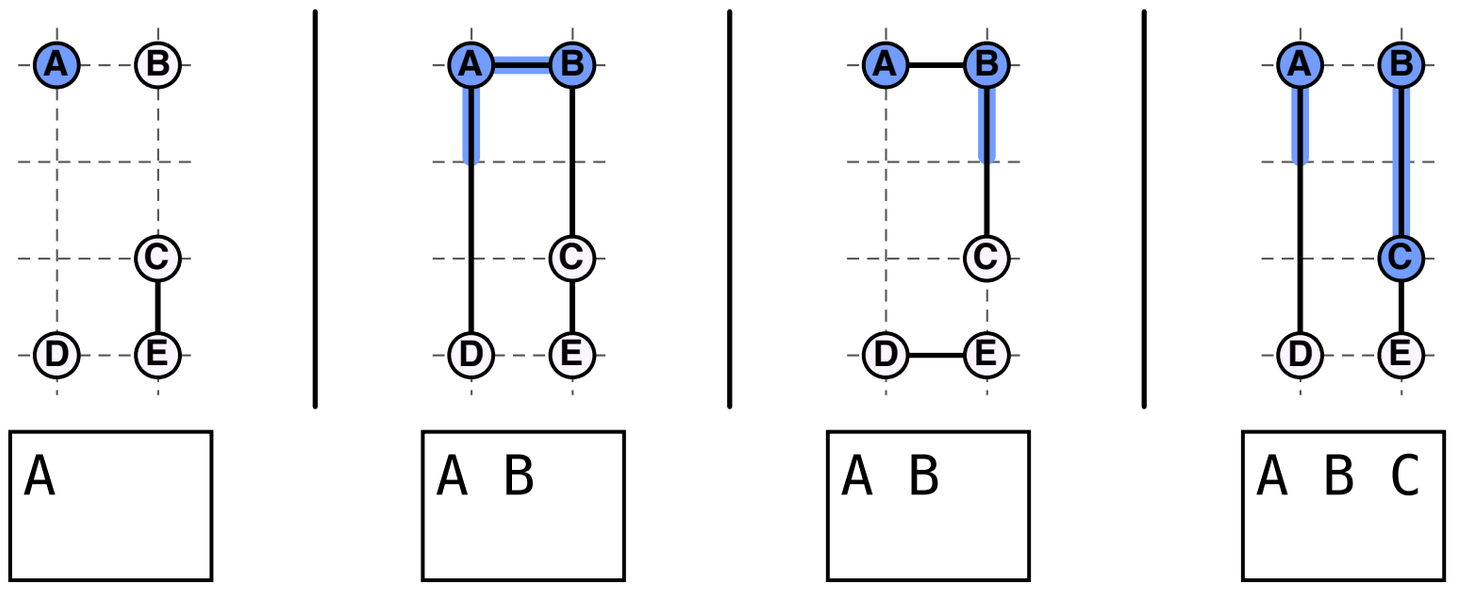

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Spatio-temporal Paths

Spatio-temporal path:

$$(v_0, t_1), (v_1, t'_{\text{arr}_1}), (v_2, t'_{\text{arr}_2}), \ldots, (v_n, t'_{\text{arr}_n})$$

Properties:

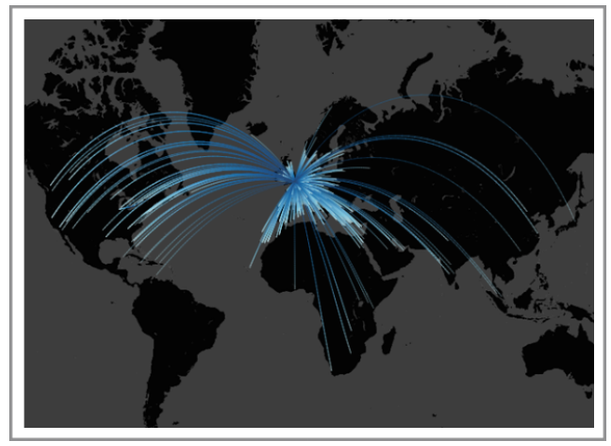
- **Latency**: time to reach destination from source
- **Spatial length**: overall physical distance travelled
- Number of hops

Shortest spatio-temporal path:

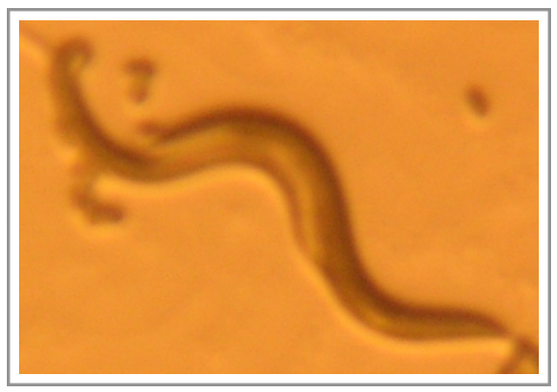
- (1) Minimum latency, and (2) Minimum spatial length

Measurement on real-world Networks

London Underground
Passenger Transit (270 stations)

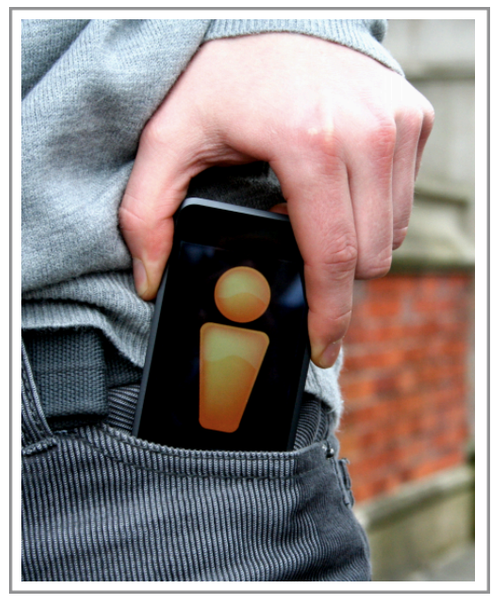


US Domestic Flights
Passenger Transit (299 Airports)



C. Elegans (Nematode)
Neural Network (279 neurons)

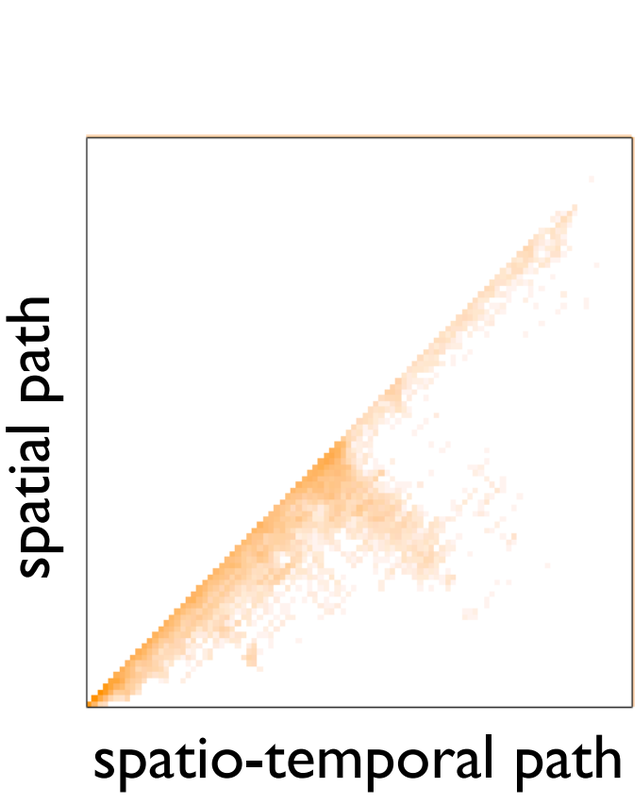
StudentLife
Mobile Comms
(Calls & SMS Logs)
(22 Dartmouth Students)



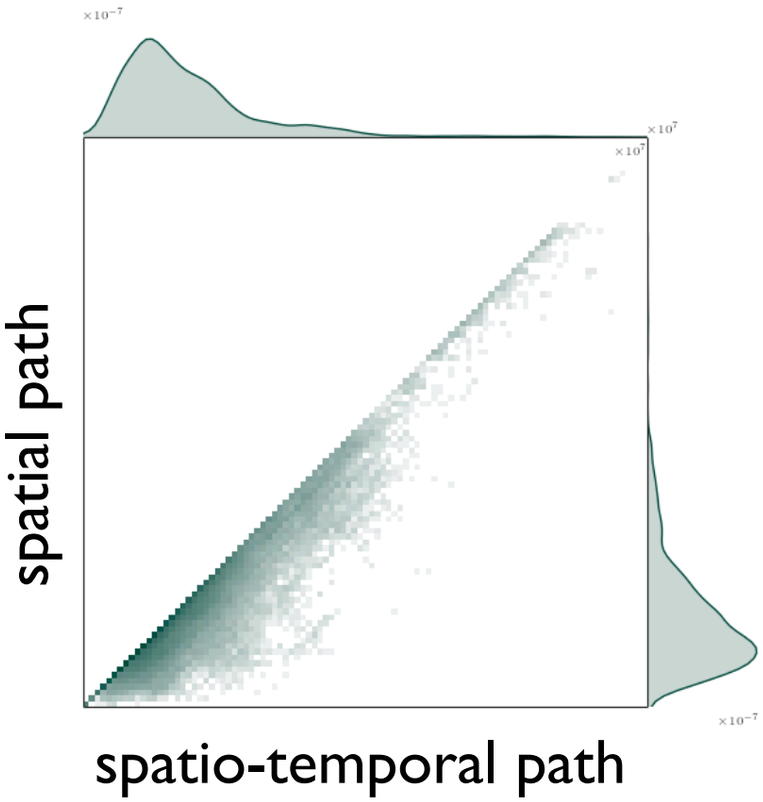
Real-world Networks

	Propagation Type	Nodes	Edges (Aggregate Network)	Time-Varying Topology	Mobile Nodes	Median Propagation Speed
Underground	Passenger Transit	270	628	✔	×	8 m/s
Flights (U.S. Domestic)	Passenger Transit	299	3947	✔	×	152 m/s
C. Elegans (Neural Network)	Synaptic Transmission	279	2990	×	×	0.44 mm/s
StudentLife (Mobile Comms)	Phone Calls & SMS	22	68	✔	✔	instantaneous

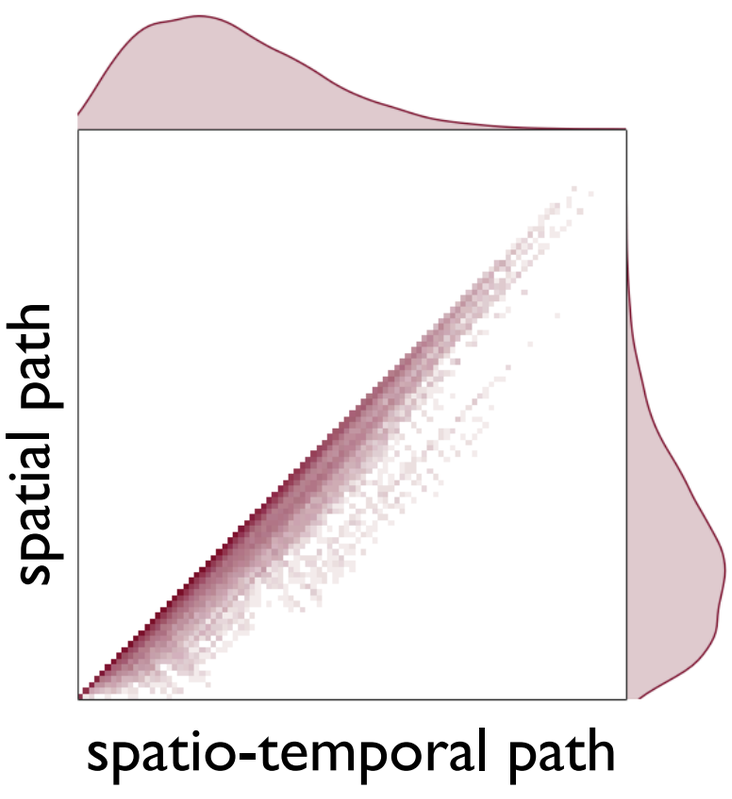
Spatial Shortest Paths vs Spatio-Temporal Shortest Paths: Spatial Distance



celegans



flights



underground

temporal shortcuts → spatial detours

Measuring robustness of spatio-temporal networks

Robustness of Spatio-Temporal Networks

- How does the system respond to node failure?
  - Failure: Node deactivation
  - The behaviour of a spatio-temporal network can be measured in terms of its **topological**, **temporal**, and **spatial** structure

Measures of Performance

Giant strong component size
Largest number of mutually reachable nodes

Relative loss in temporal efficiency
Temporal efficiency: *Average reciprocal* ***temporal*** *distance*
Lower efficiency means more "delay" in the network

Relative loss in spatial efficiency
Spatial efficiency: *Average reciprocal* ***spatial*** *distance*
Lower efficiency means shortest paths traverse longer distances

Measures of Performance

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Relative loss in temporal efficiency

Temporal efficiency: *Average reciprocal* ***temporal*** *distance*

Lower efficiency means more "delay" in the network

Relative change:

1 ⇒ same efficiency as intact network	0 ⇒ all disconnected

Relative loss in spatial efficiency

Spatial efficiency: *Average reciprocal* ***spatial*** *distance*

Lower efficiency means shortest paths traverse longer distances

Relative change:

1 ⇒ same efficiency as intact network	0 ⇒ all disconnected

Node Failure: Random

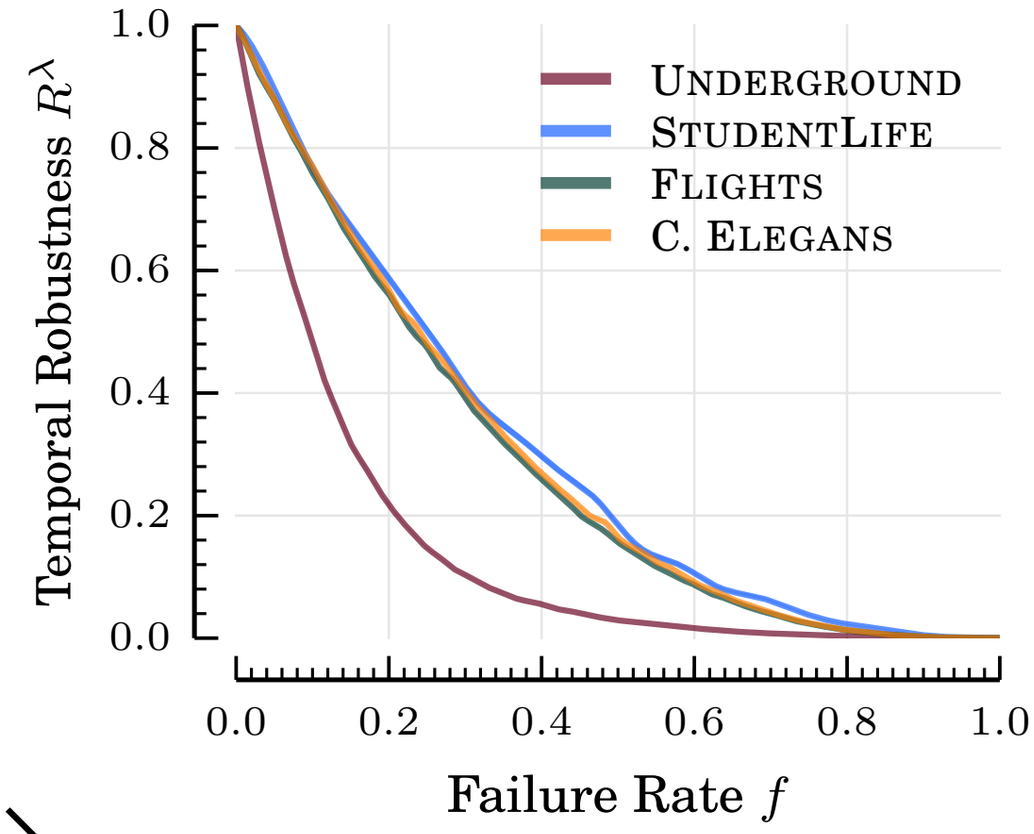
Random failure

Node deactivated with failure probability *f*

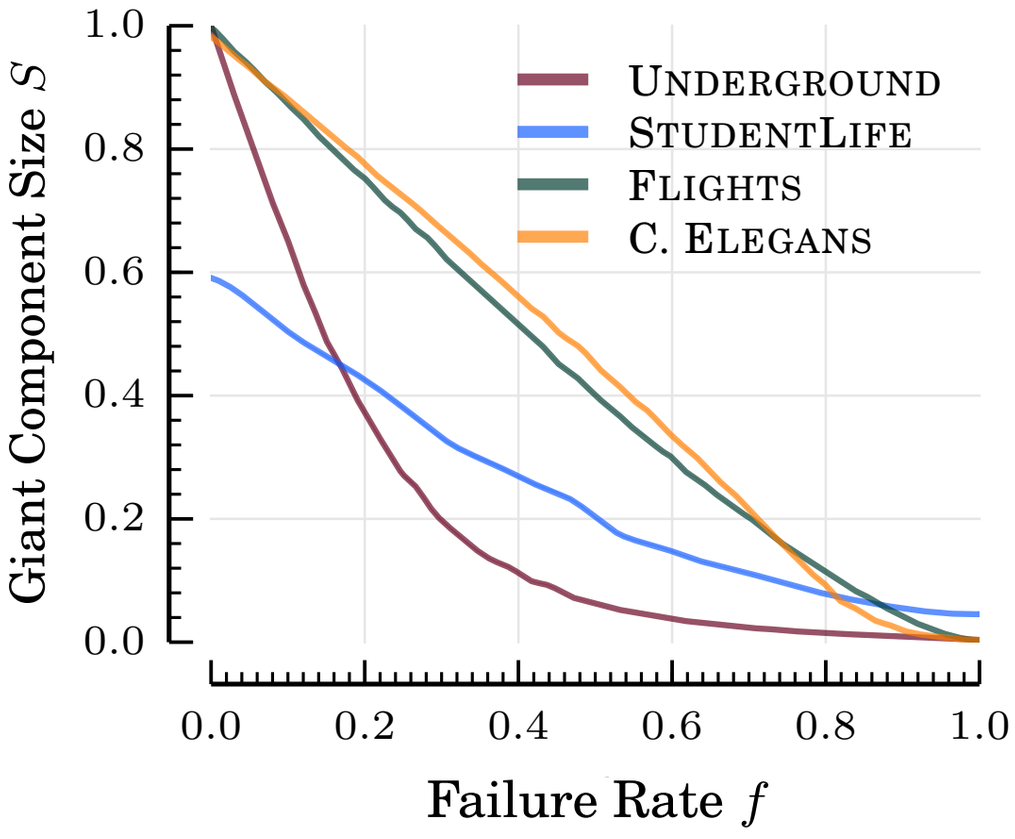
Rand.

Resilience to Random Failure

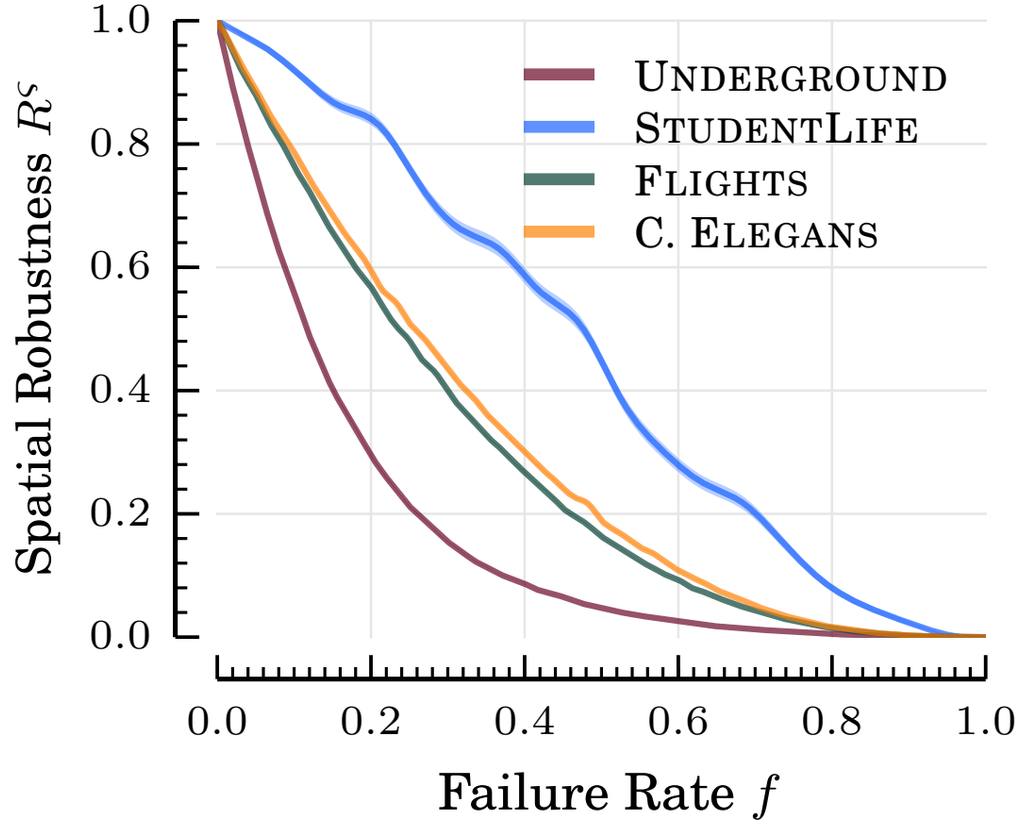
Temporal →



Reachability
(Giant Temporal Comp.)
↓



Spatial ↘



Resilience to Random Failure

Temporal →

Reachability
(Giant Temporal Comp.)
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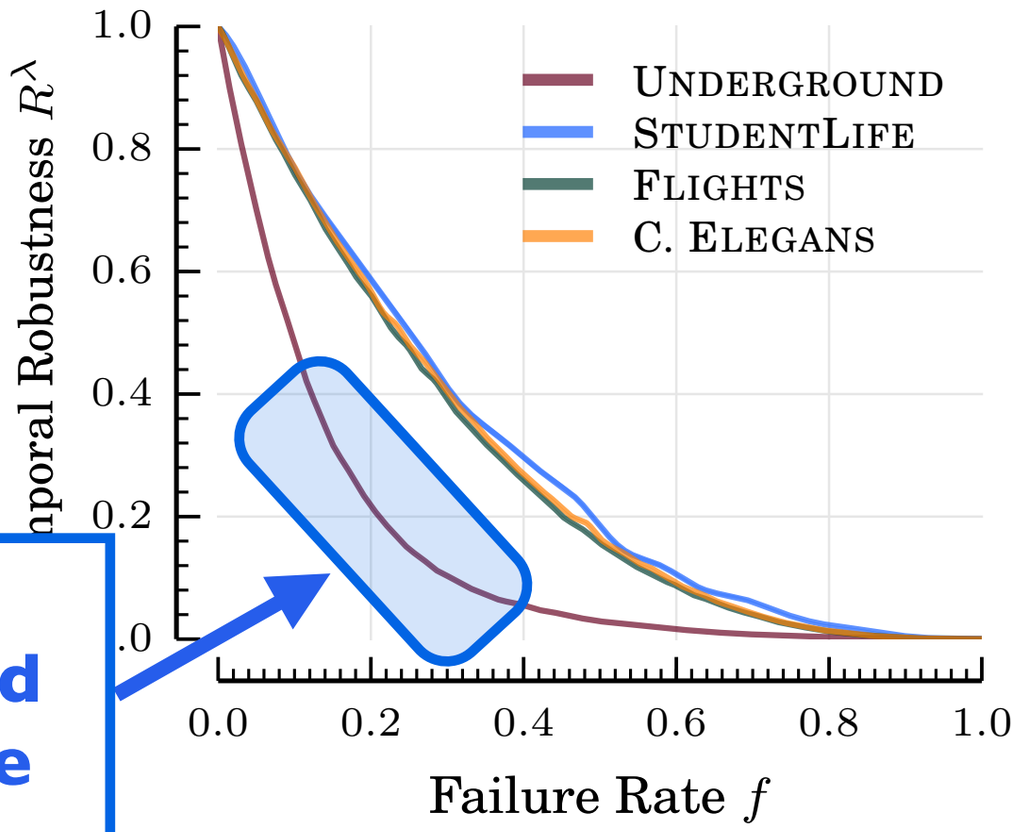
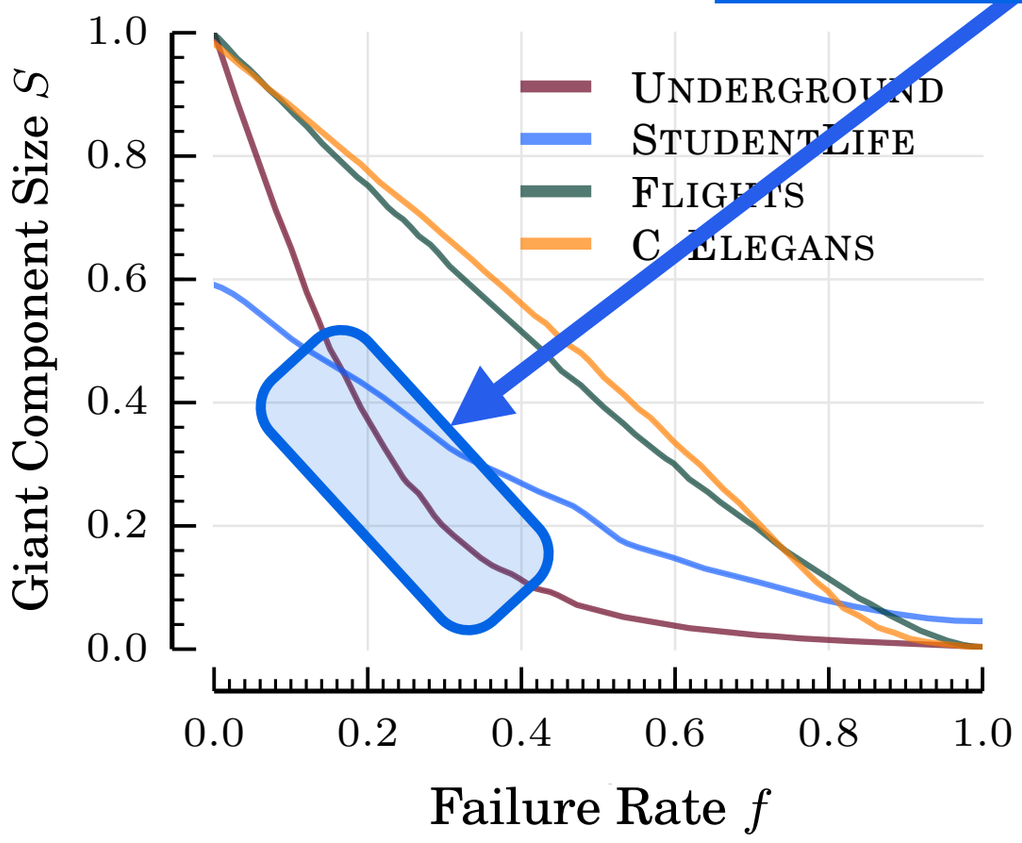
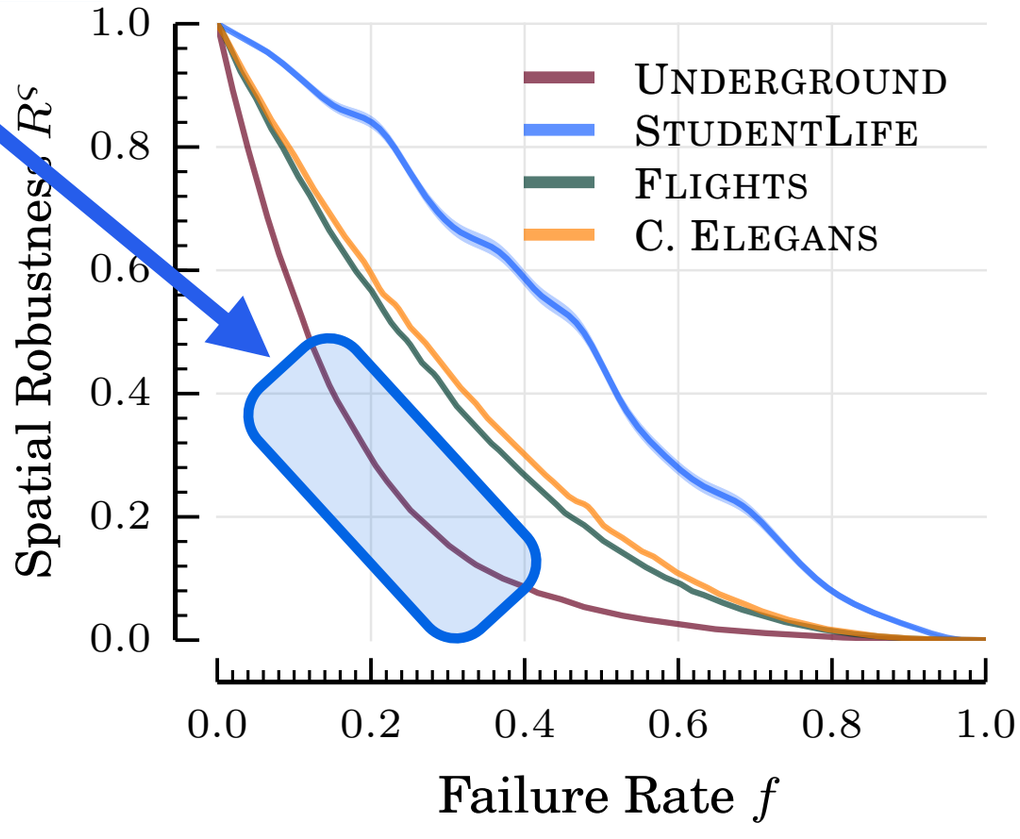


Underground highly fragile





Node Failure: Systematic

Random failure — Rand.

Node deactivated with failure probability *f*

Systematic attacks

Path betweenness: — PB

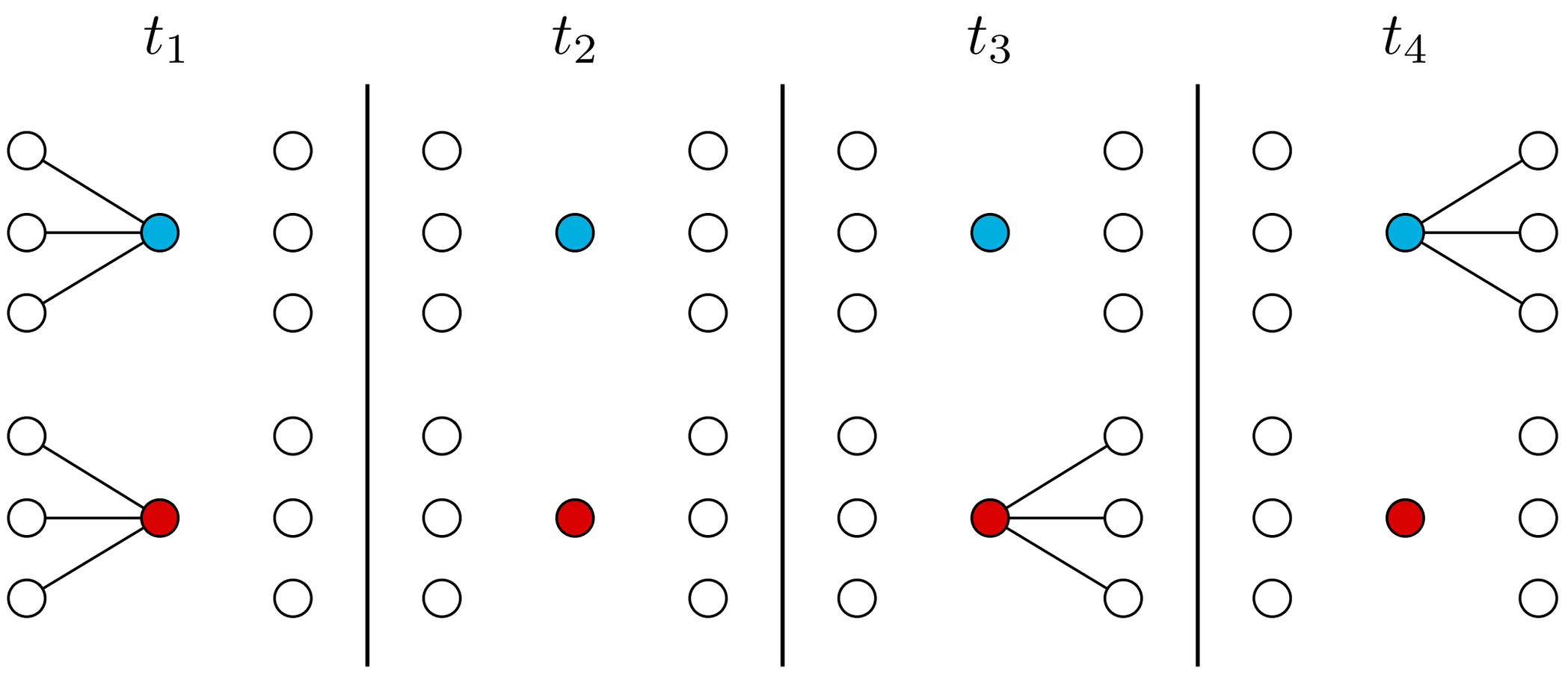
Target nodes which **support many shortest paths**
Preferentially dismantle the giant component

Betweenness efficiency: — BE

Target nodes which allow **rapid information flow**
Preferentially degrade the temporal efficiency; i.e., increase delay in the network

(Very effective attacks. Worst case behaviour. Require global knowledge.)

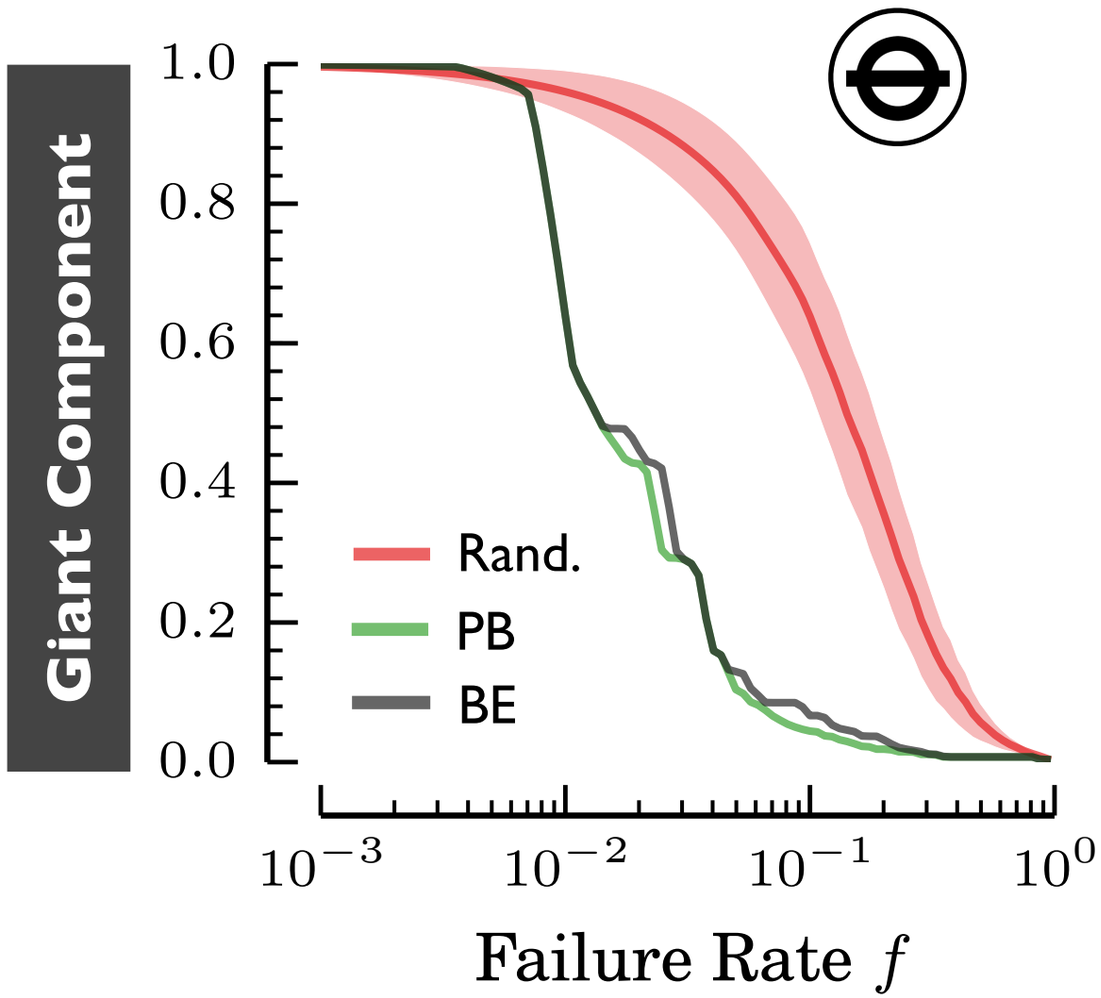
Node Failure: Systematic

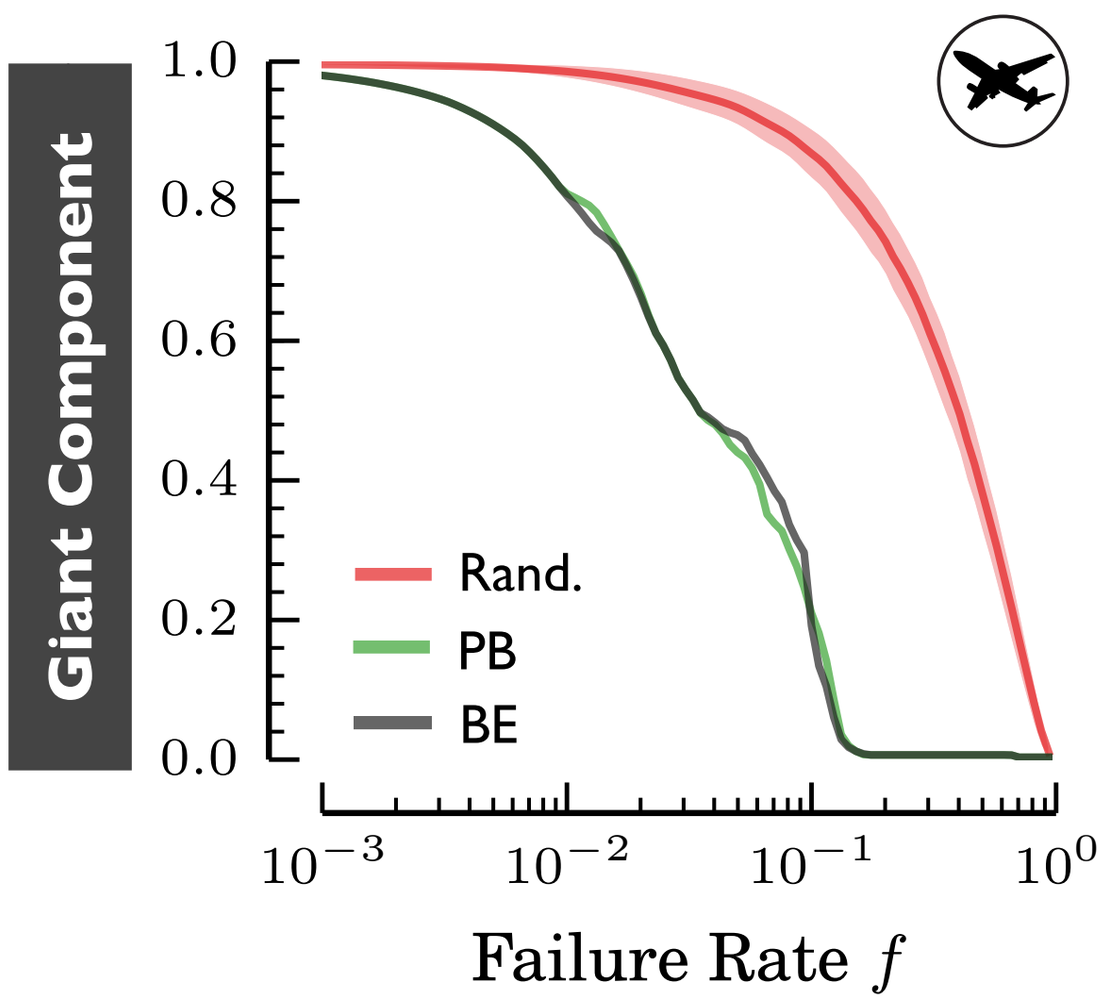


both have same path betweenness

● has higher temporal betweenness efficiency

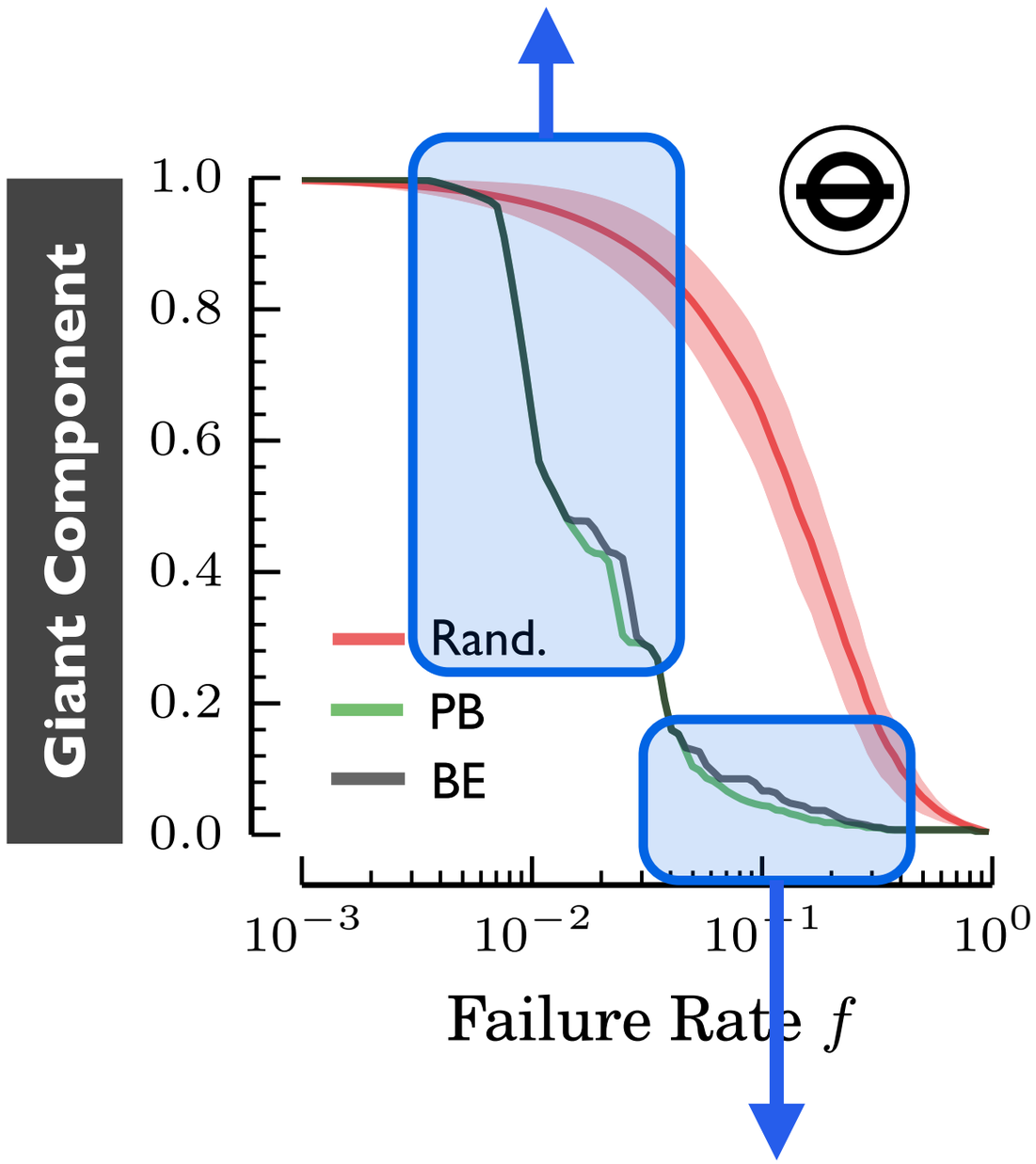
Attack Tolerance: Giant Component

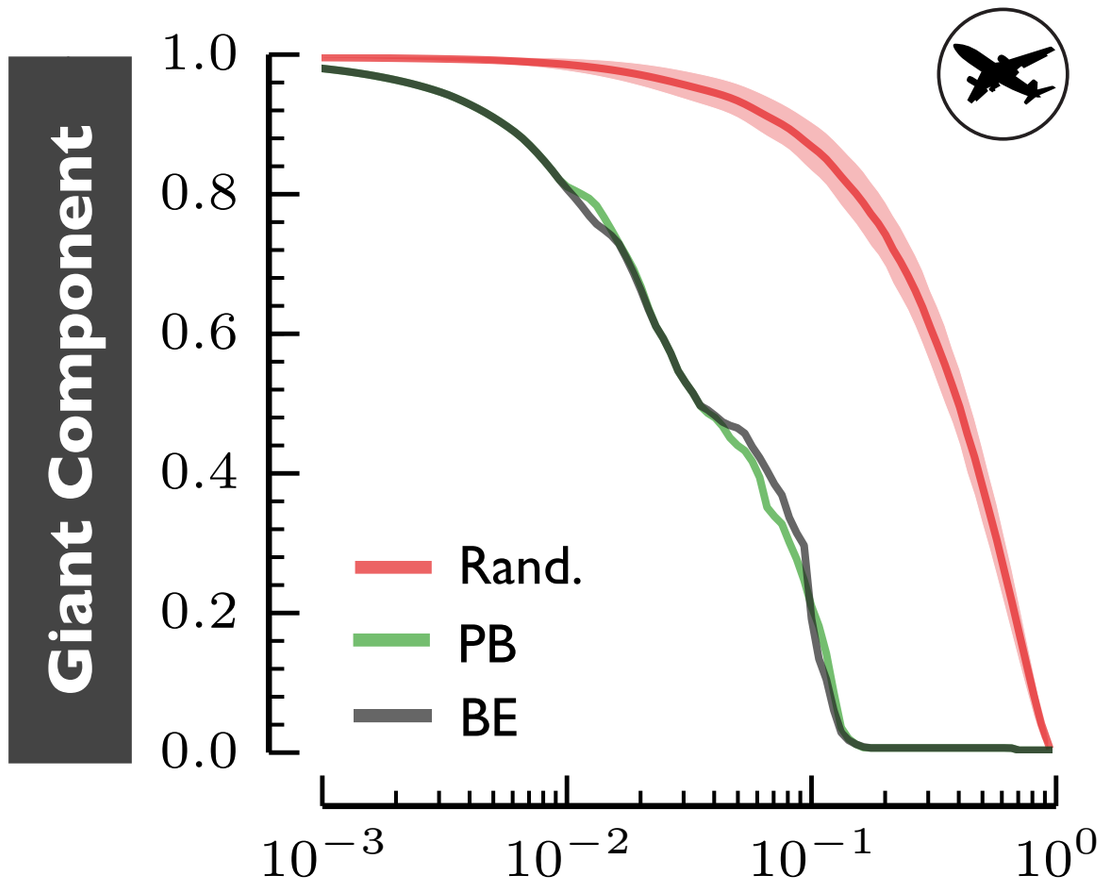




Attack Tolerance: Giant Component

peripheries (total = 190 nodes) rapidly disconnected within 13 deactivations (f<4%)





f = 5% to 45%
resilient central region

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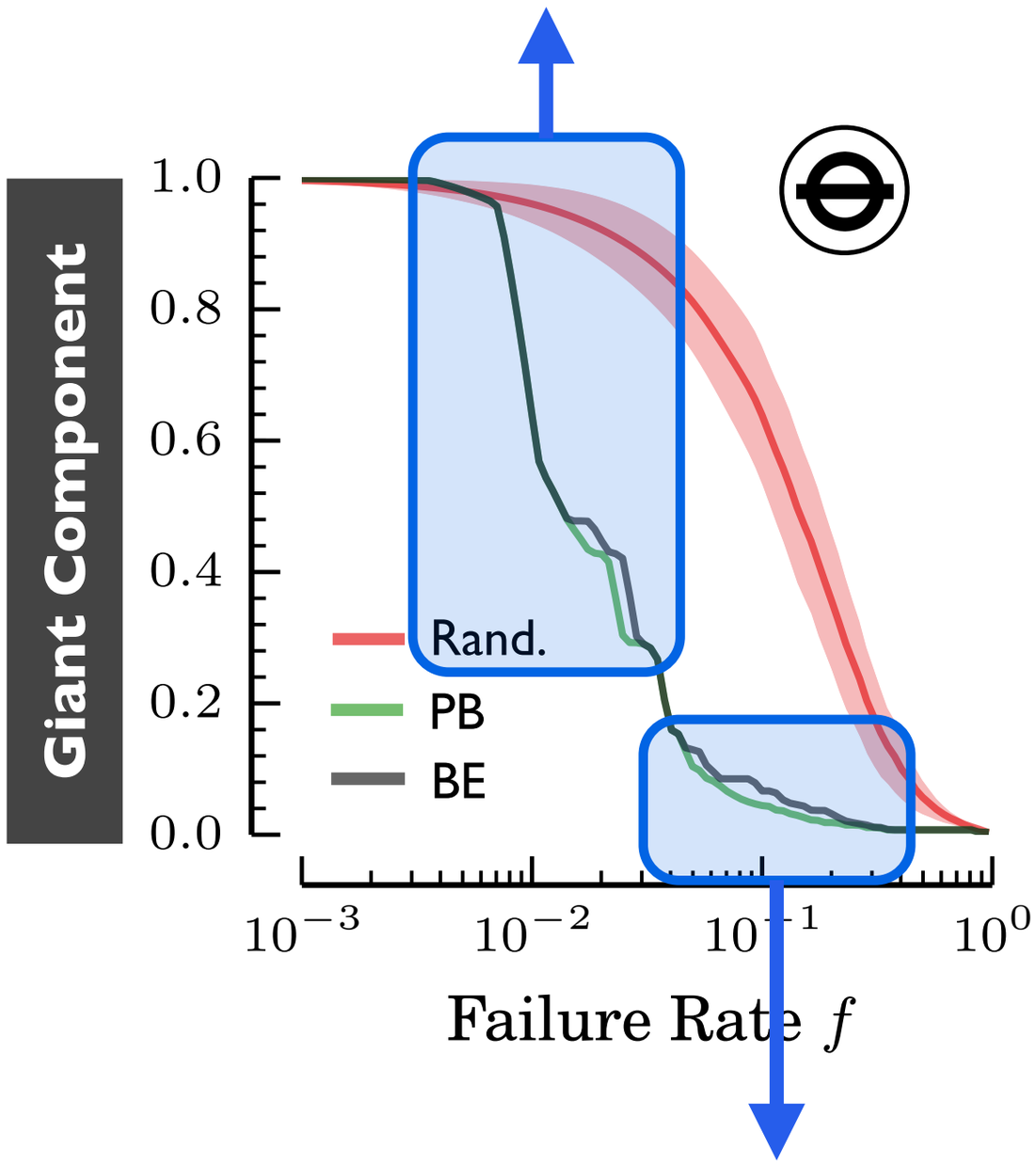

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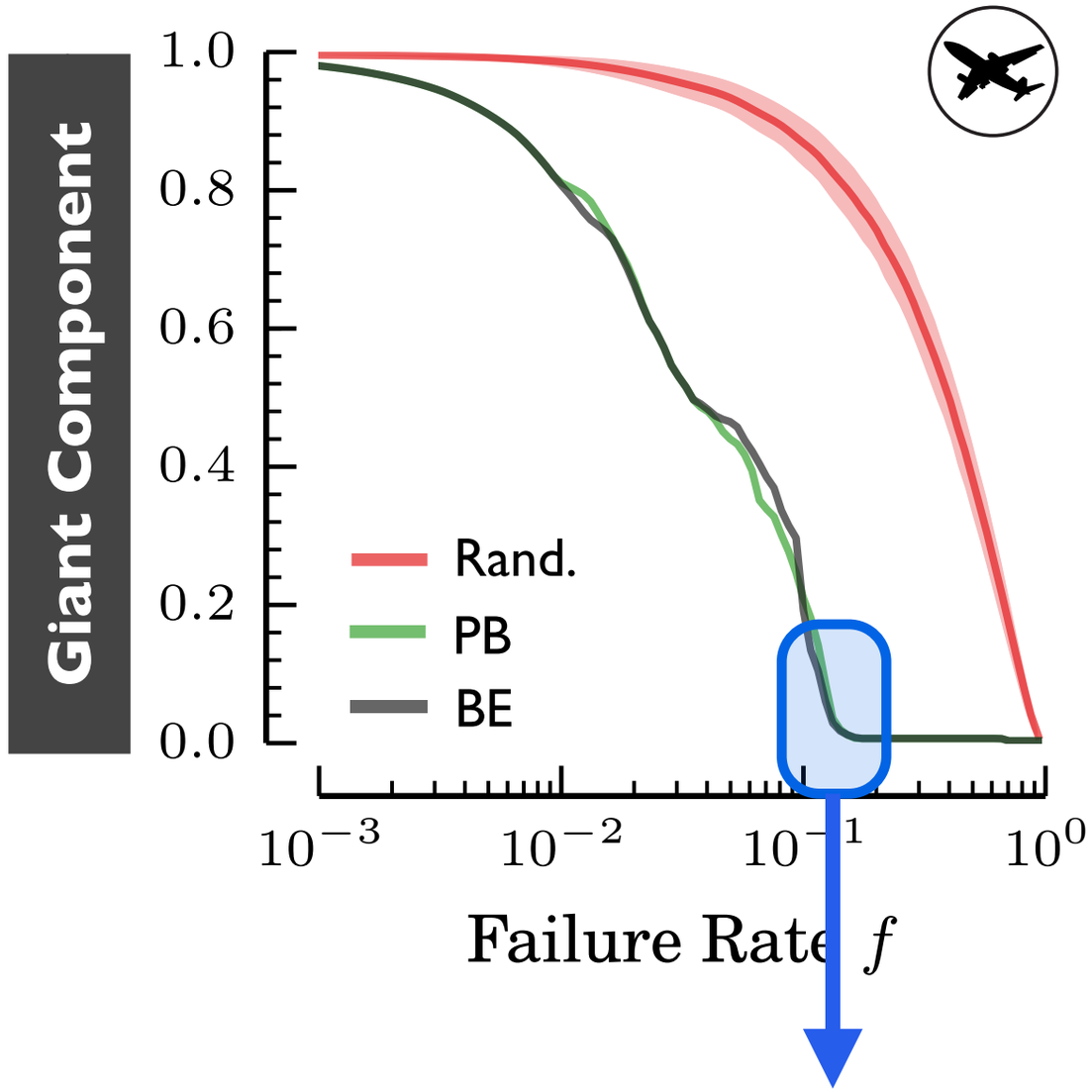
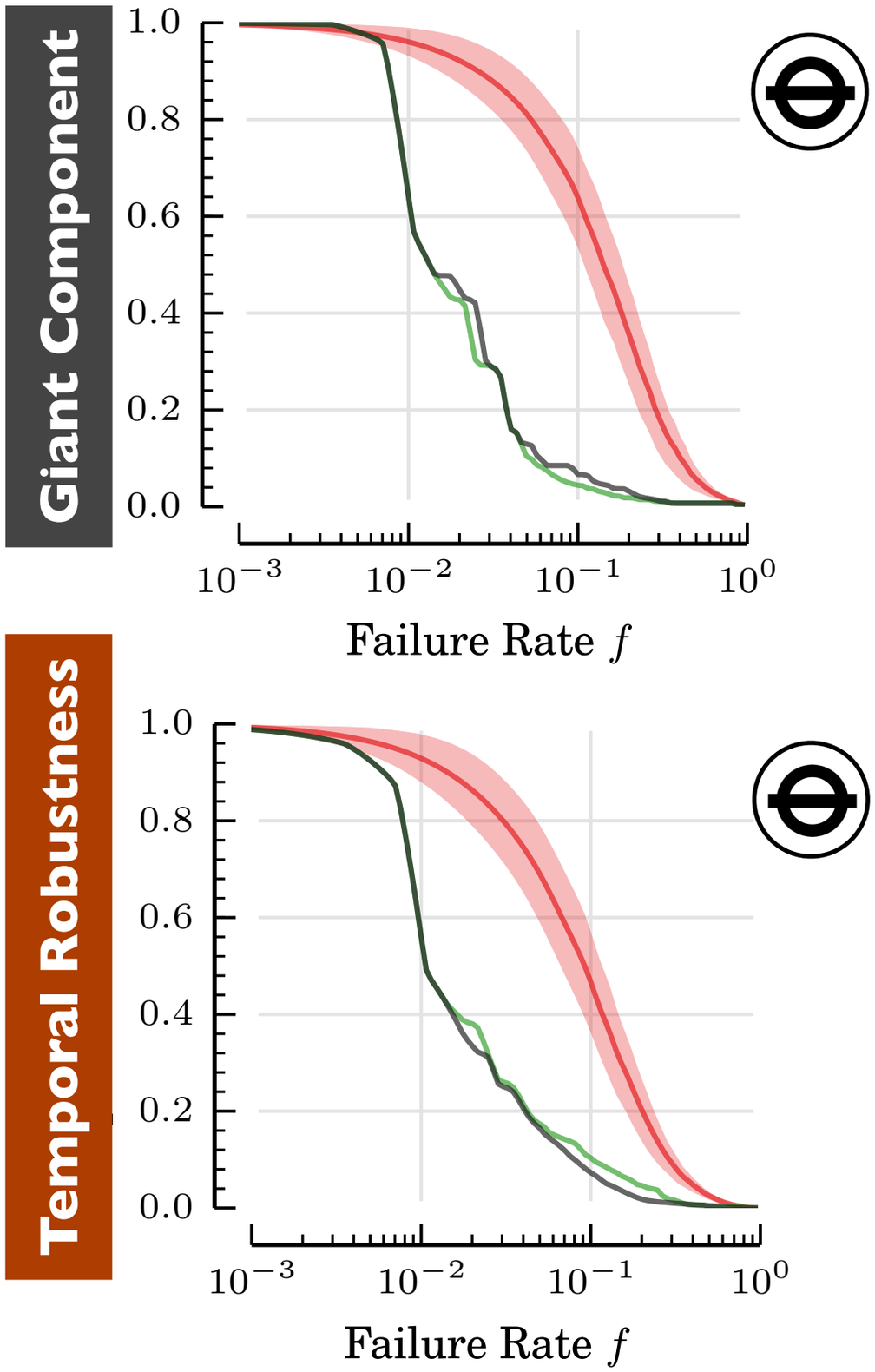
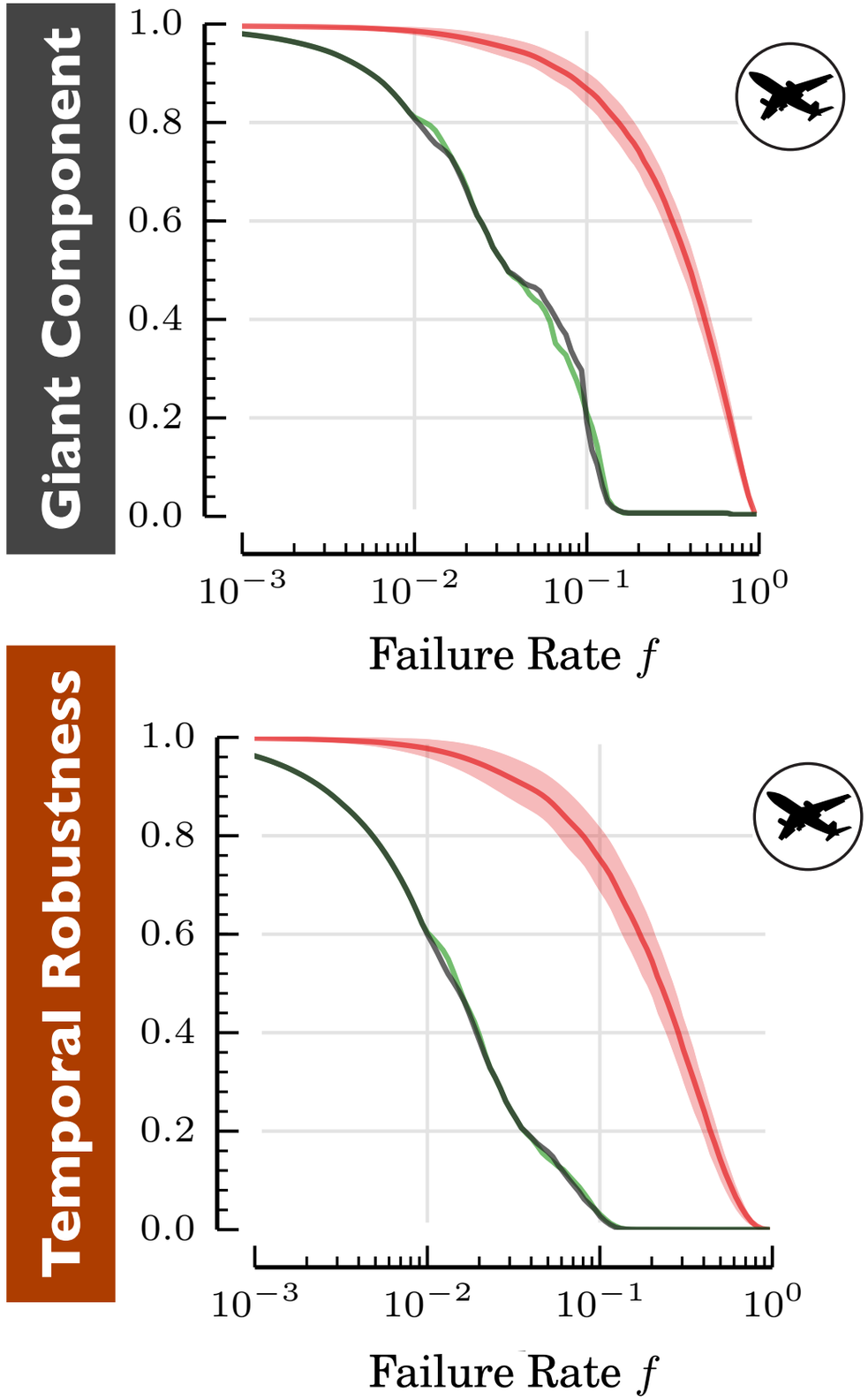


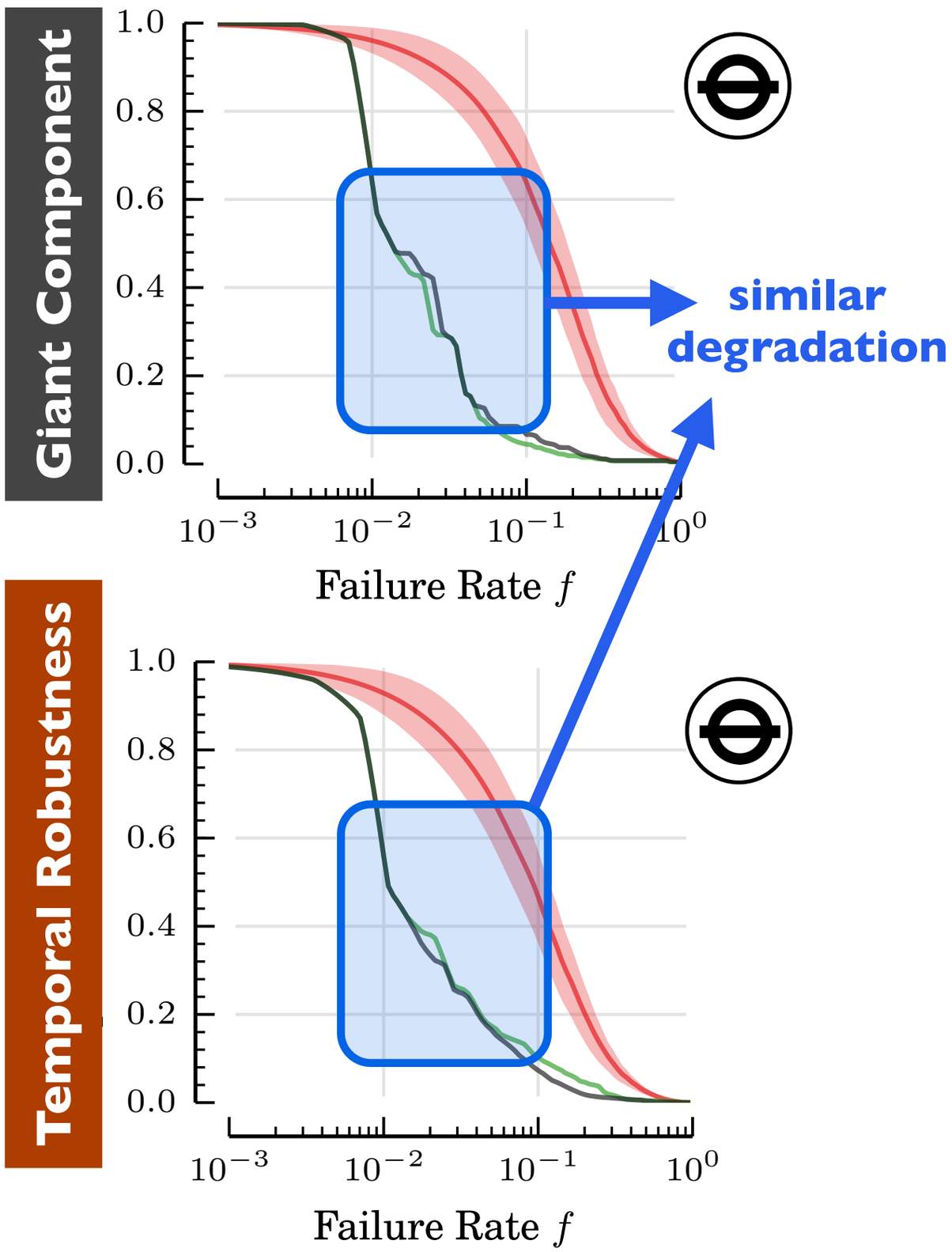
f = 16%

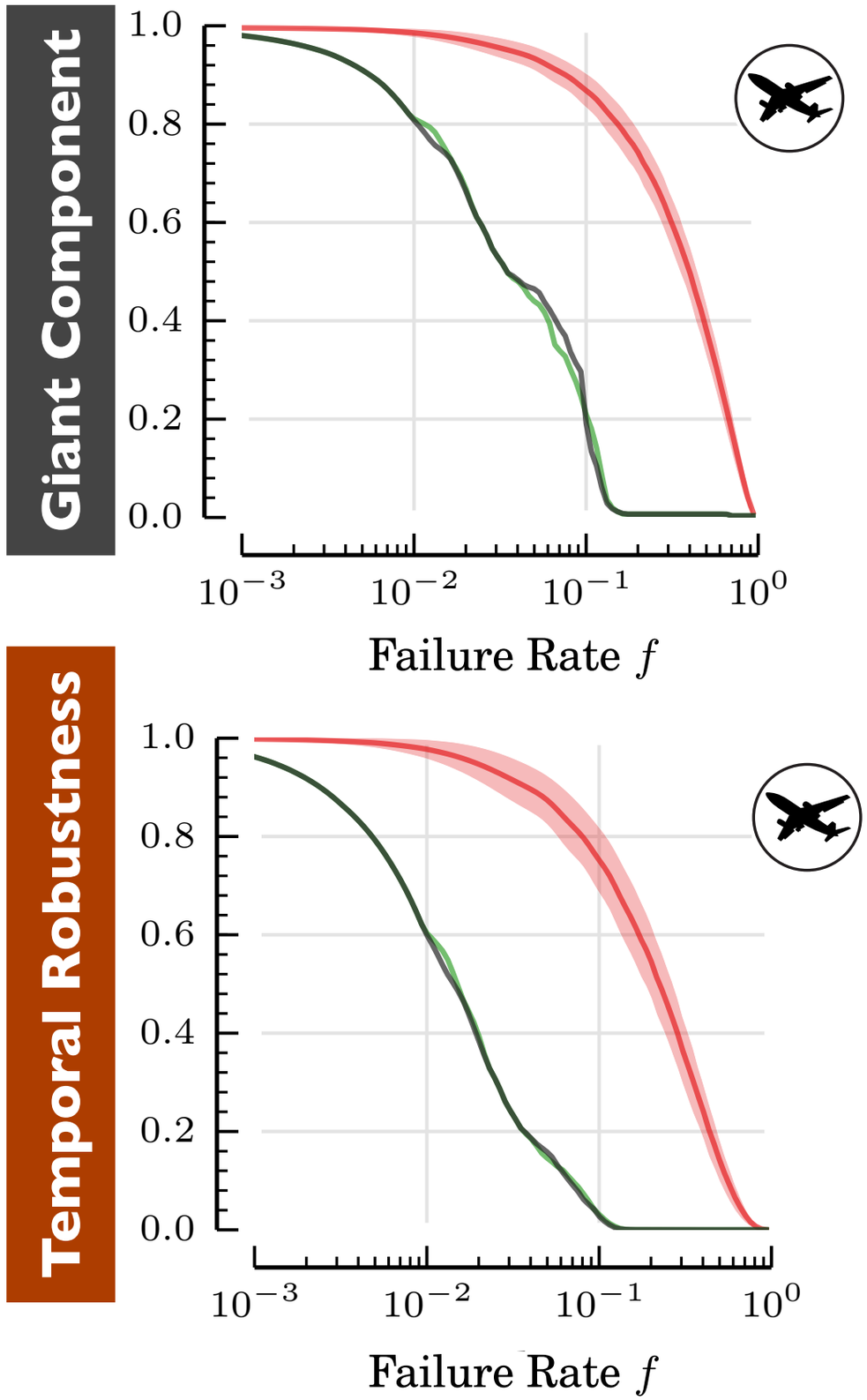
Giant Component vs Temporal Efficiency



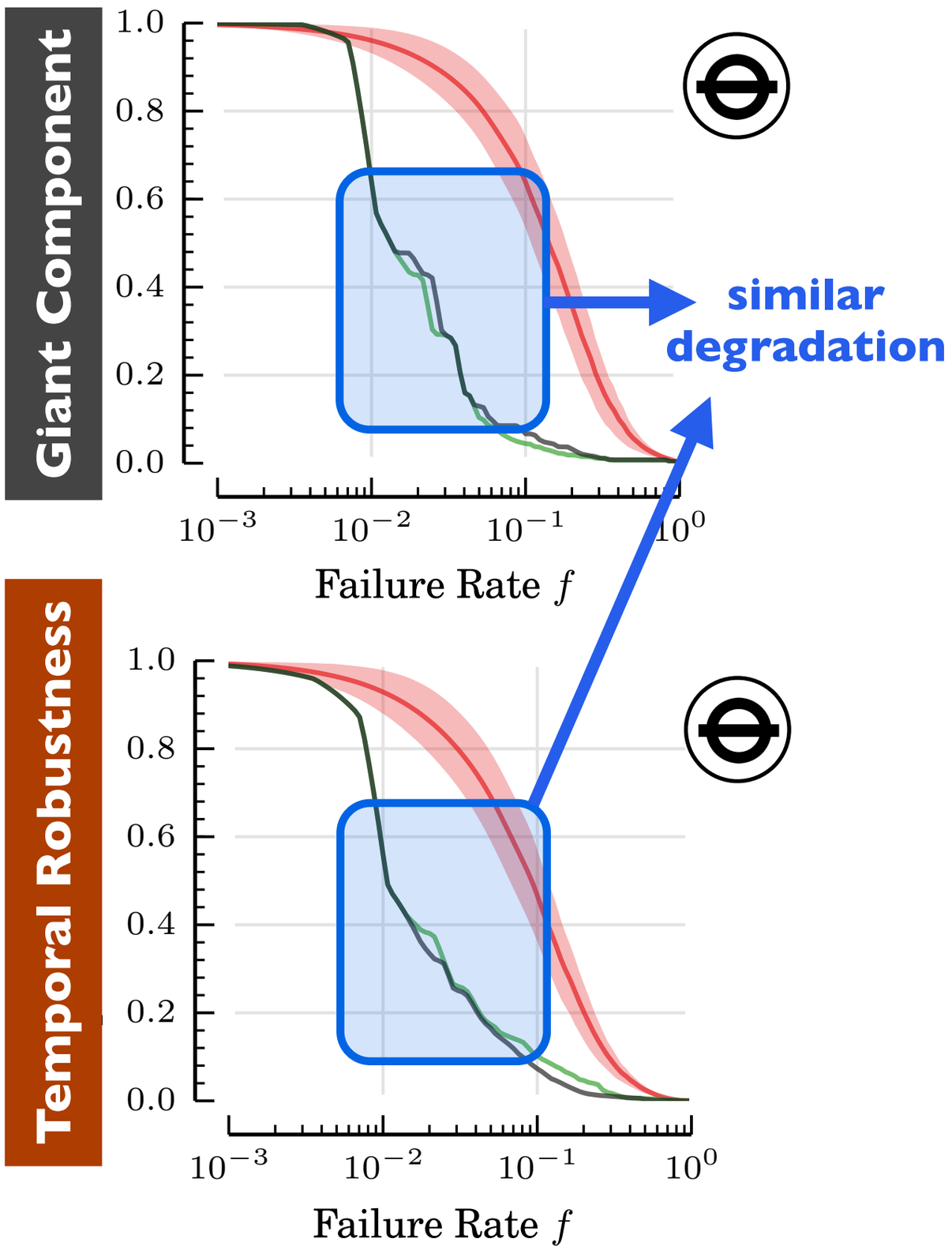


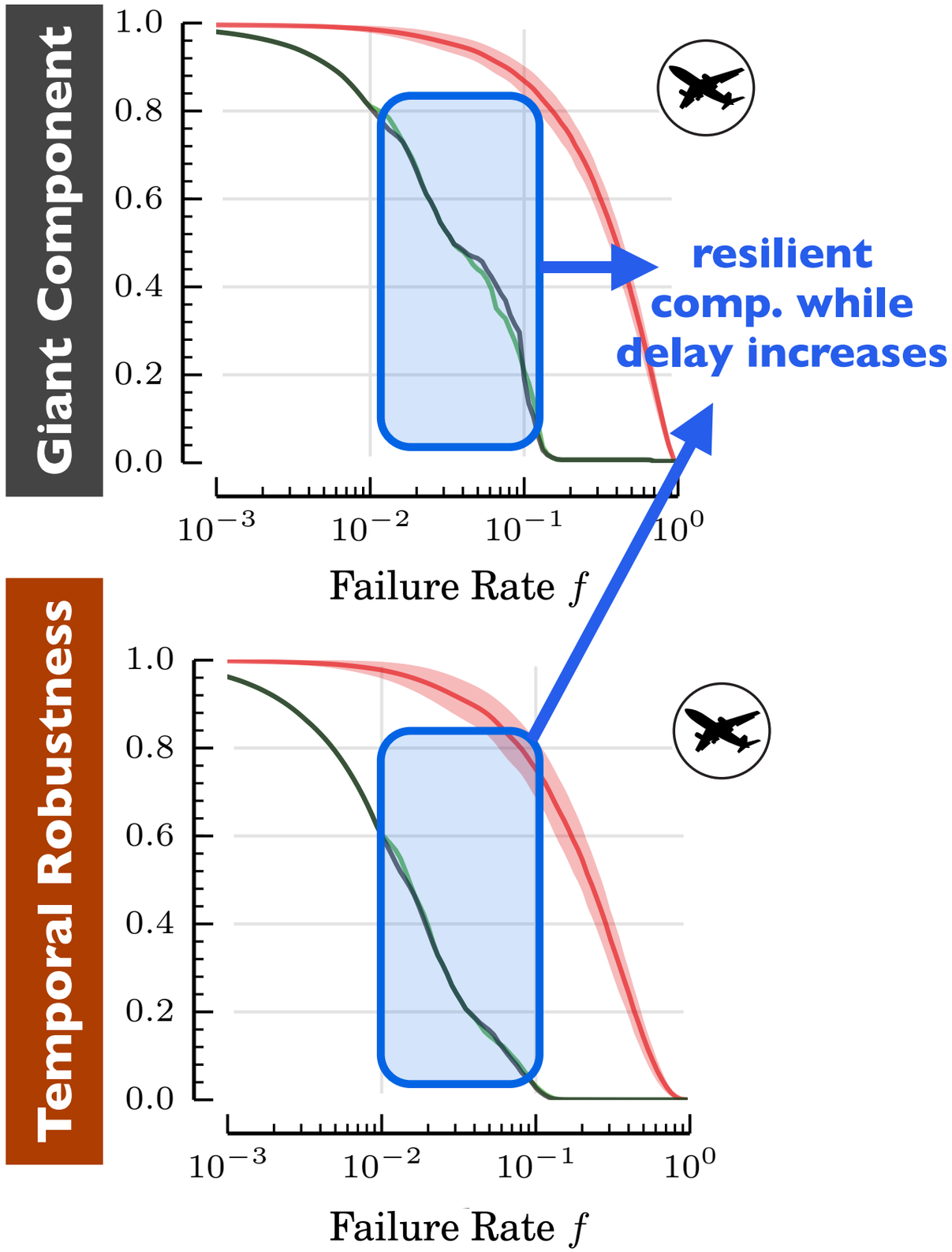
Giant Component vs Temporal Efficiency



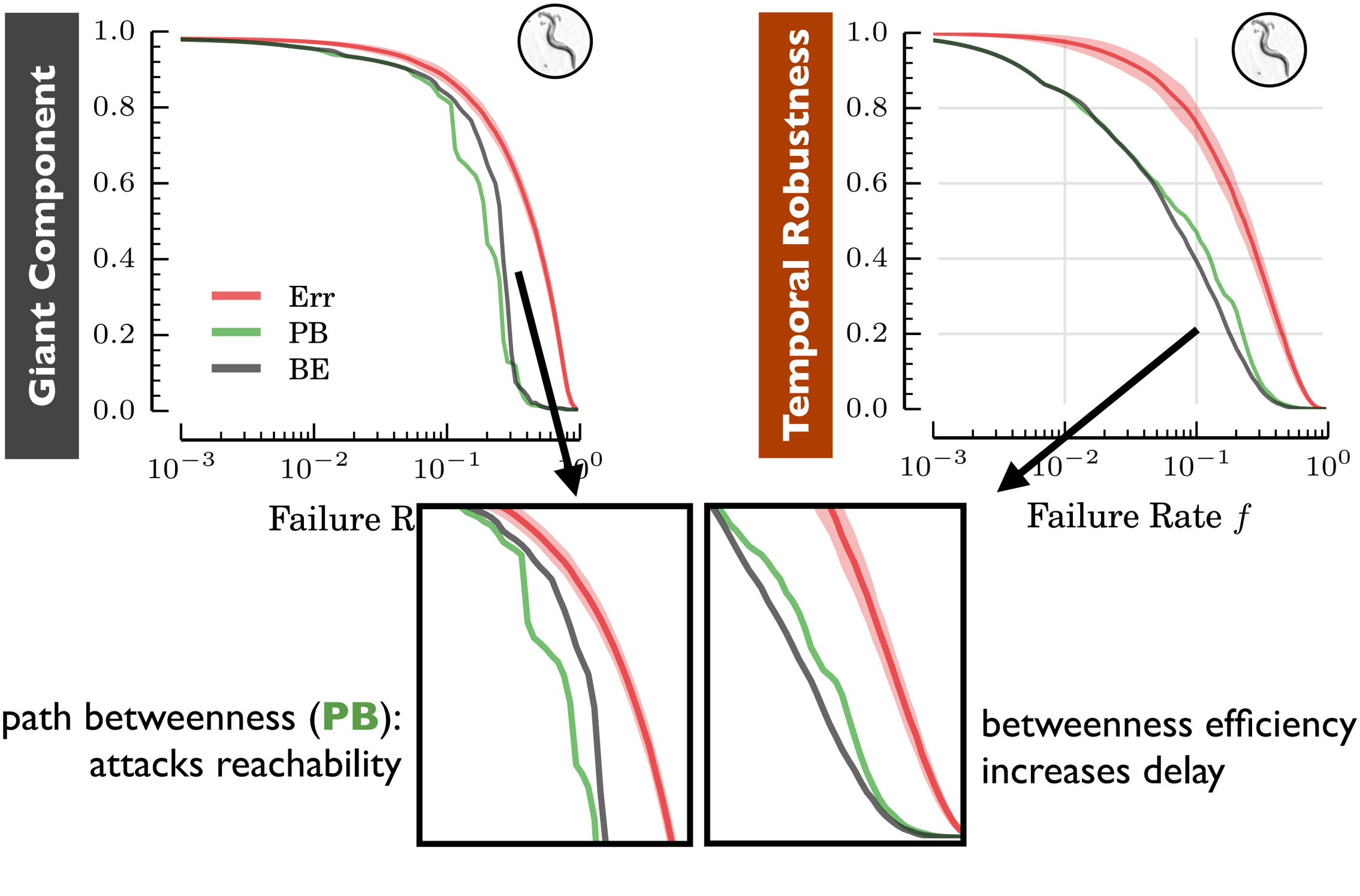


Giant Component vs Temporal Efficiency





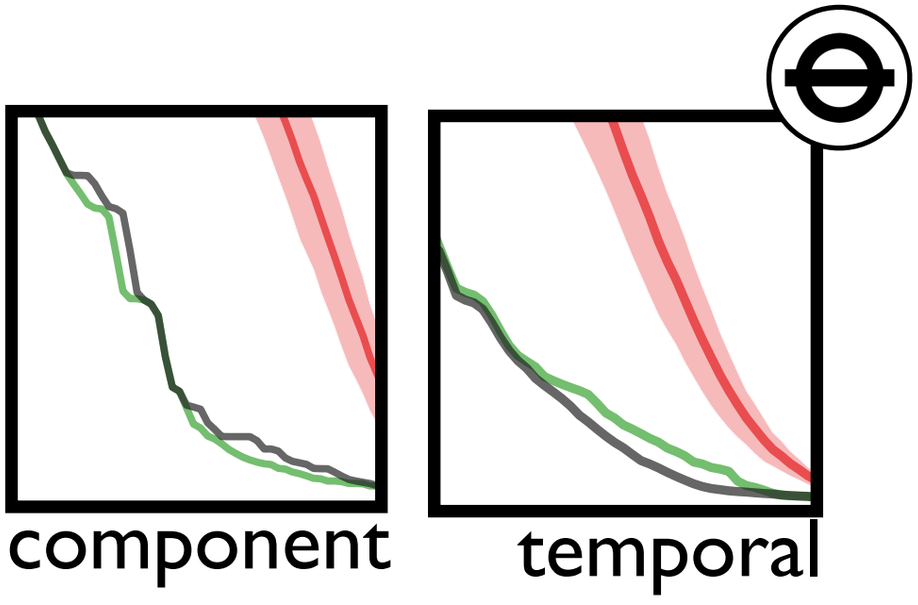
Attacks on Giant Component and Temporal Efficiency

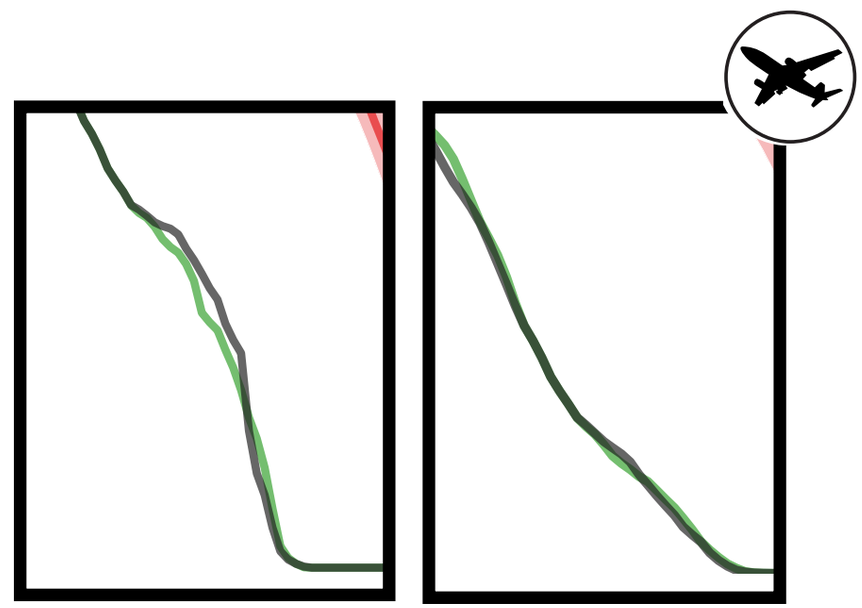


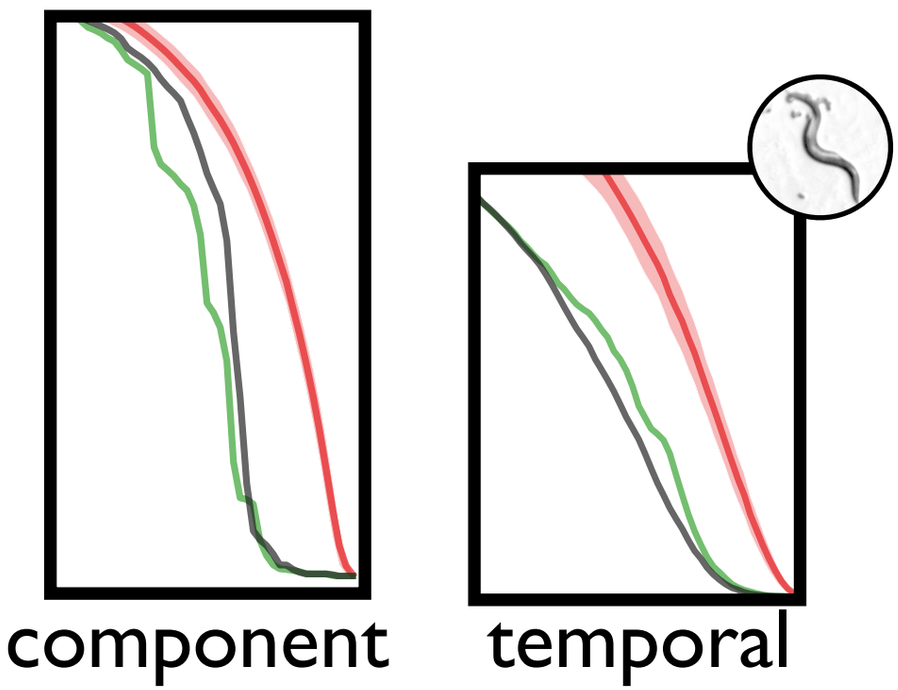
path betweenness (**PB**):
attacks reachability

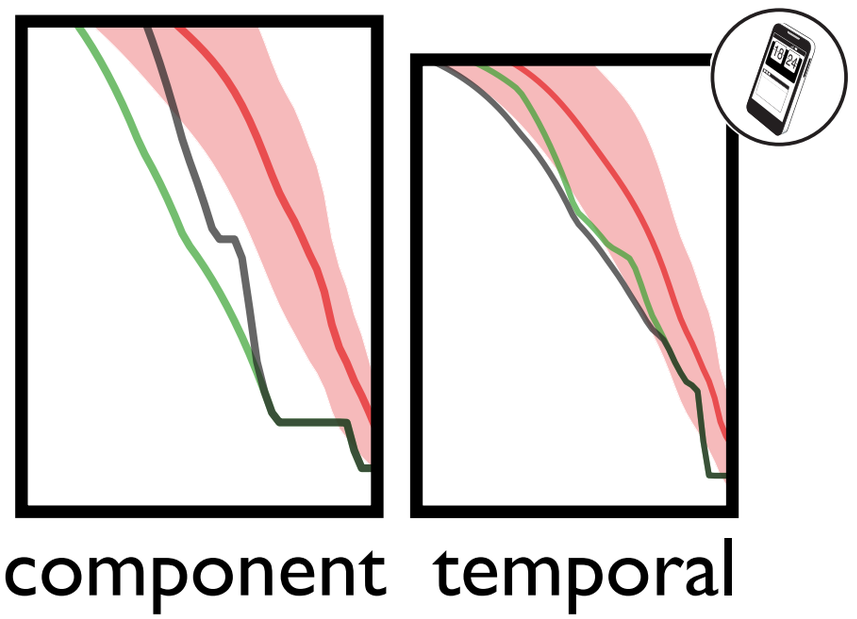
betweenness efficiency (**BE**):
increases delay

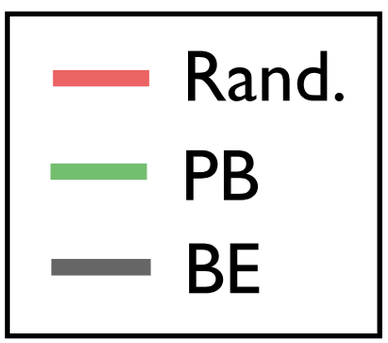
Attacks on Giant Component and Temporal Efficiency



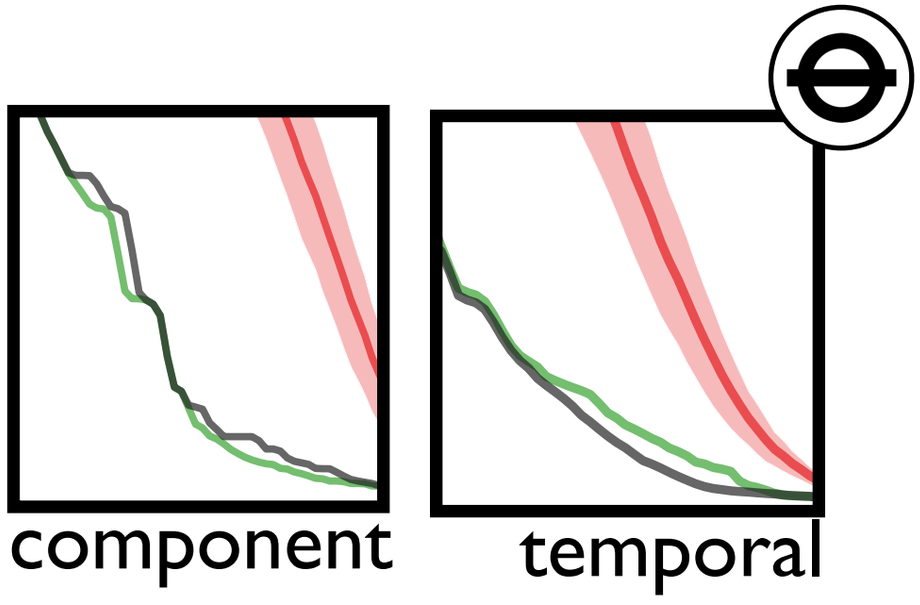


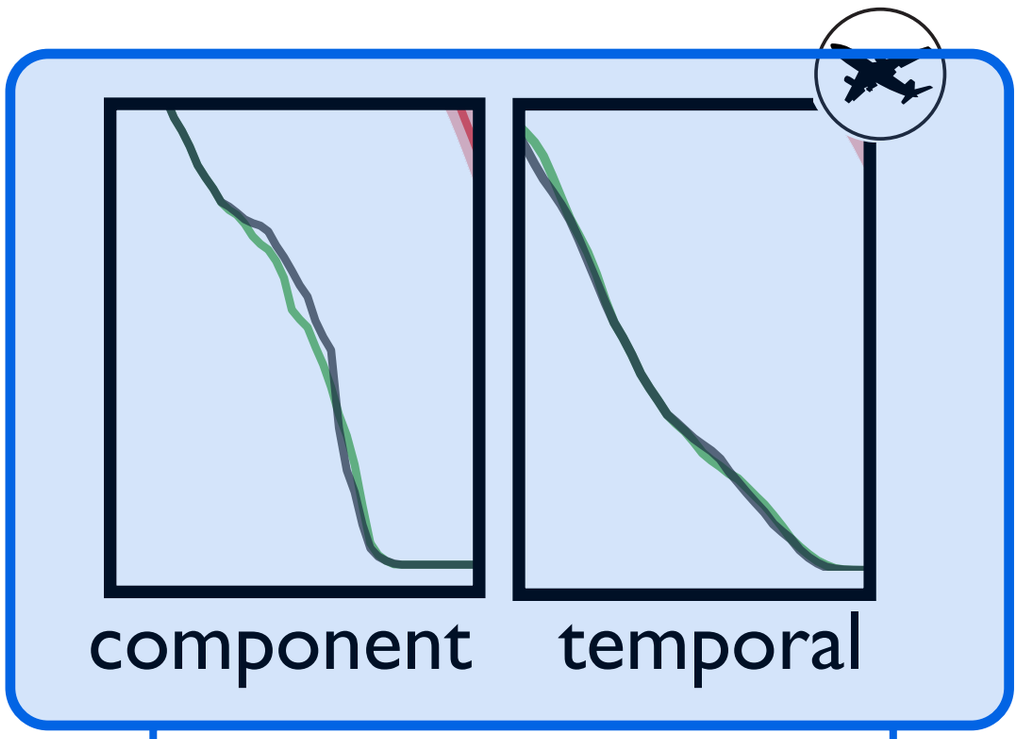


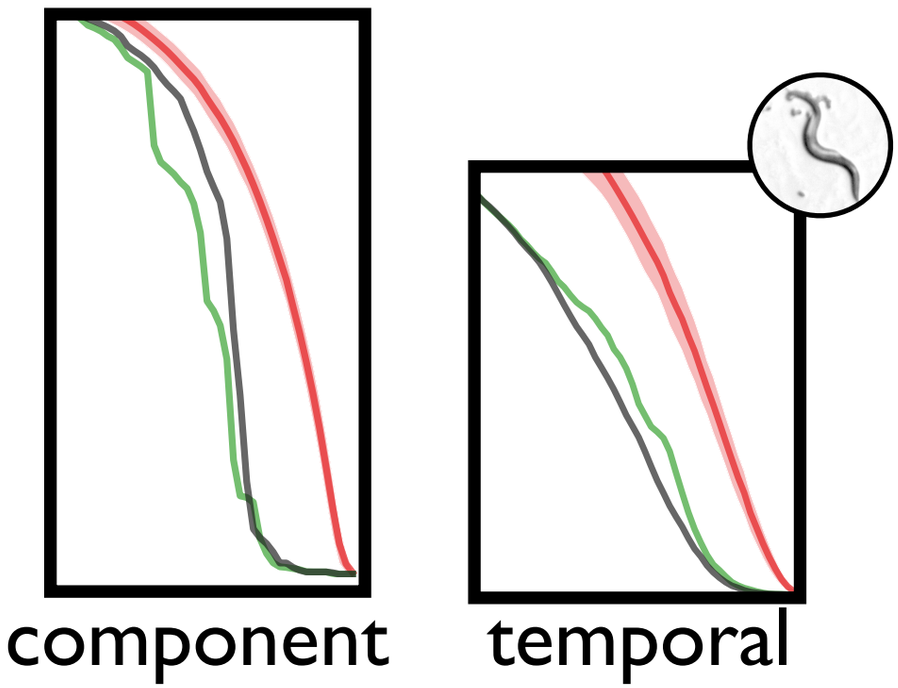




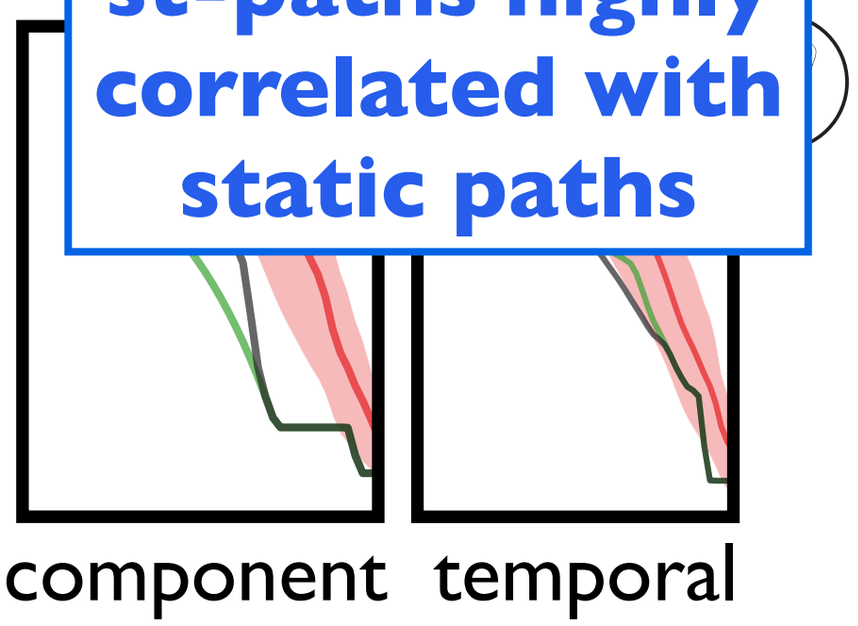
Attacks on Giant Component and Temporal Efficiency

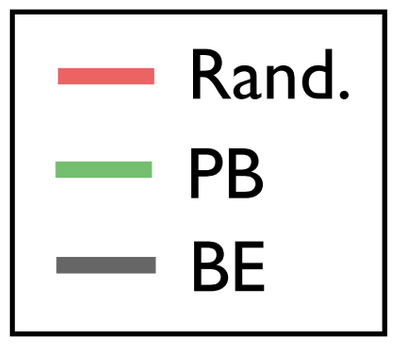






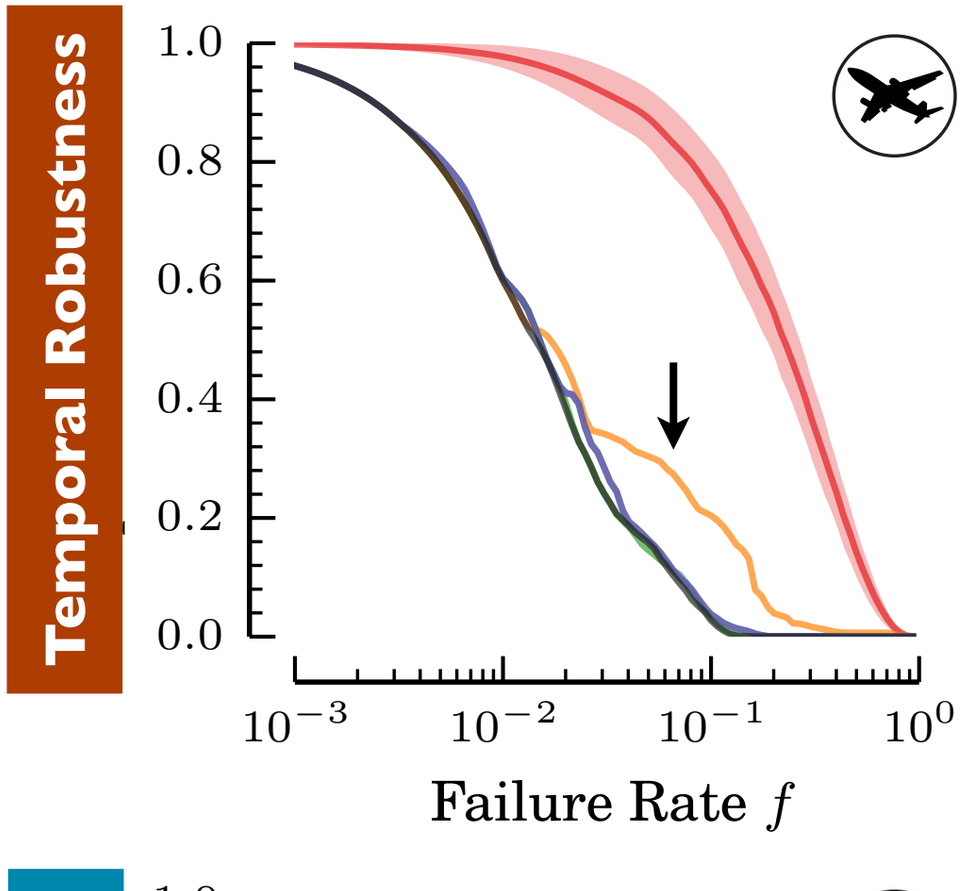
st-paths highly correlated with static paths

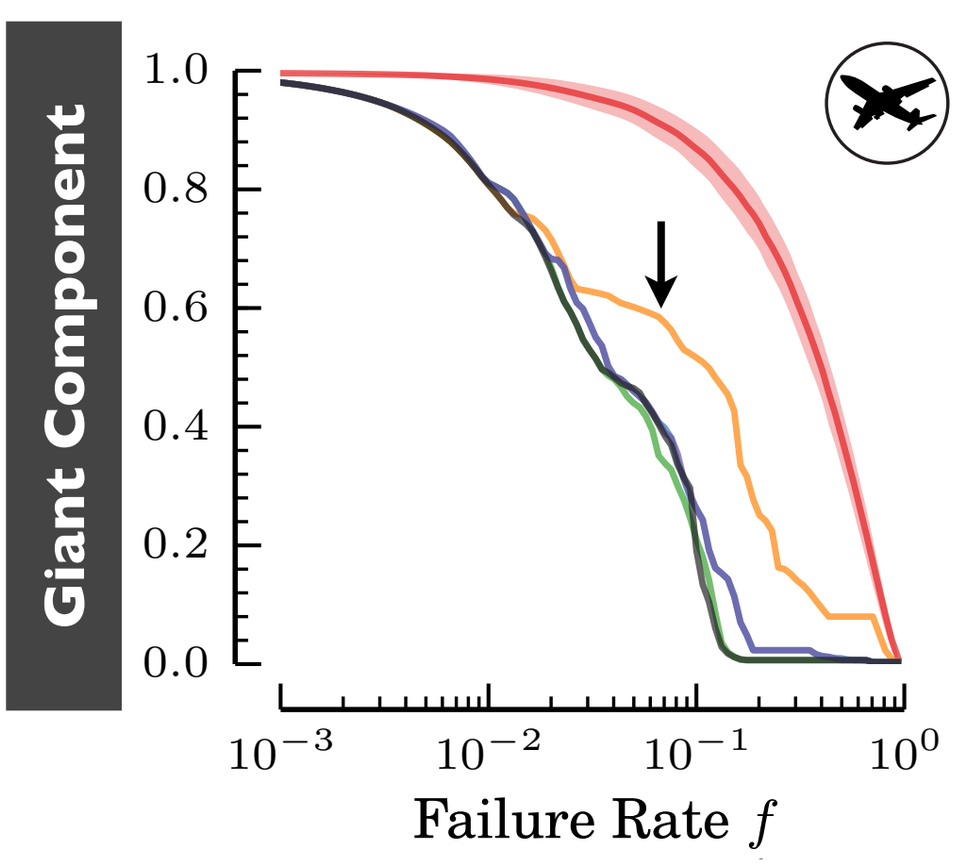


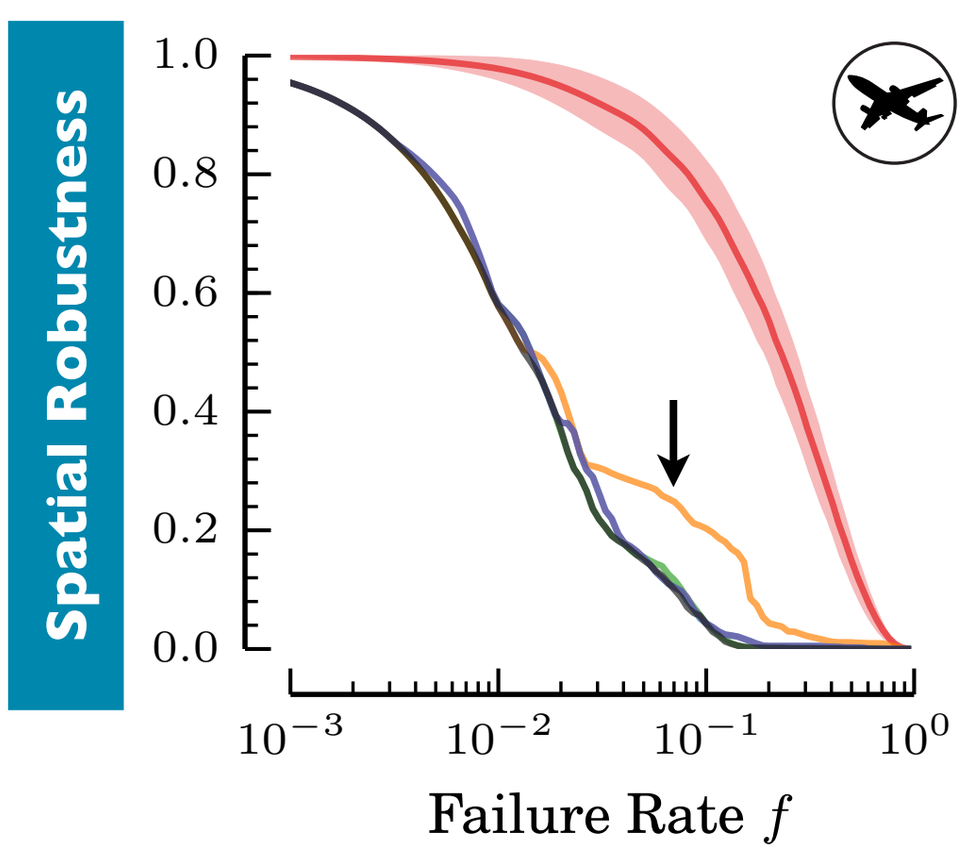


Temporal Closeness Attack?

e.g., in Flights network







Summary I

- Framework for modelling spatio-temporal systems as networks
- Generalisation of temporal networks with **spatially embedded nodes** and **paths that preserve space-time constraints**
- Avoids over-simplification due to **aggregation** (static network models) and **instantaneous transmission** (temporal network models)

Summary II

- Systematic attacks can be designed to target different aspects of a network; e.g., **topological** (reachability) vs. **temporal** structure
  - **Path betweenness attack** – dismantles the giant component
  - **Betweenness efficiency attack** – increases delay

Ongoing Work

- Relationship between underlying topology vs propagation speeds (shortcutting effects)
- Synthetic temporal network models
- Empirical disruptions – real-world regimes of random failure / preferential attack
- Localised failures

There are worse signalling stations to accidentally flood with concrete...

Random Removal
f = 6 / 270

Temporal Robustness
94%

Jan 2014
6x stations closed

Temporal Robustness
89%

Worst-Case
(BE Attack)

Temporal Robustness
32%

Thanks for listening!

Spatio-Temporal Complex Networks: Reachability, Centrality, and Robustness

http://arxiv.org/abs/1506.00627

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LASAGNE
Cutting through complexity

SEVENTH FRAMEWORK PROGRAMME

Attribution

Globe

"Earth - Illustration". DonkeyHotey (Flickr CC). May 2011.
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C. Elegans

"I: these are nematodes". snickclunk (Flickr CC). July 2006.
https://www.flickr.com/photos/snickclunk/209926410

Roulette Wheel

"roulette". eatsmilesleep (Flickr CC). August 2011.
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